UNIT II
AC CIRCUITS

GENERATION OF ALTERNATING VOLTAGES & CURRENTS

Alternating voltages may be generated by rotating a coil in a magnetic field & by rotating a magnetic field within a stationary coil.

The value of the voltage generated depends, in each coil, upon the no of turns on the coil, strength of the field & the speed at which the coil or magnetic field rotates.

Equations of alternating voltages & Currents:

$E(t) = E_m \sin\theta = E_m \sin\left(\frac{2\pi}{T}\right)t$

$I = I_m \sin 2\pi f t = I_m \sin\left(\frac{2\pi}{T}\right)$

T = time – period of alternating voltage or Current = 1/f

$\therefore$ Induced emf varies as sine function of the time angle wt & when emf is plotted against time, a curve simulated below is obtained.

Thus curve varies in this manner is known as sinusoidal.

Terms used for periodic functions

1. **Alternating quantity: -** It is one which is changing w.r.t time e.g., V(t) , i(t), p(t)

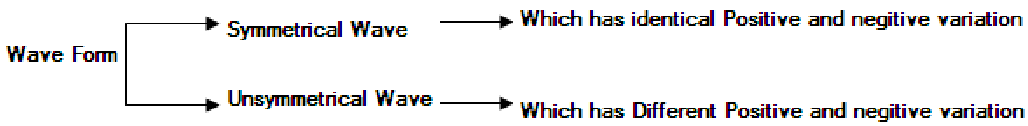
   It is period i.e. its variation repeats with certain periodicity.

   Direct current quantity:

   It is constant and is not changing w.r.t time e.g., V,I

2. **Wave form or wave shape:**

   The graphical representation of the variation of ac quantity w.r.t to time



3. **Cycle:** One complete set of variations of an alternating quantity.
4. **Time period (T):** It is the time taken for completing one cycle. It is expressed in
5. seconds (or) radius.

6. **Frequency (f):** The number of cycles completed in one second is called frequency & it is expressed in cycles/second or Hertz.

$$f = \frac{1}{T} \quad \text{or} \quad T = \frac{1}{f}$$

1 cycle = 2∏ radians, T sec = 2∏ radians

7. **Angular velocity:** Angular traced at in one second is called angular velocity ω

$$\frac{2\Pi}{T} = 2\Pi f \quad (\theta = \omega t \text{ at } t = 1 \text{ then } \theta = \boldsymbol{\omega})$$

8. **Amplitude:** The maximum value of alternating quantity is called an amplitude.

Root mean square value:

The R.M.S. value of an alternating current is given by that steady (dc) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time. It is also known as the effective or virtual value of th4e alternating current.

For computing the R.M.S. value, of symmetrical sinusoidal alternating currents, either mid-ordinate method or analytical method may be used, although for symmetrical but non – sinusoidal wires, the mid ordinate method would be found more convenient.

Analytical method:

The standard form of a sinusoidal alternating current is $I = I_m \sin \omega t = \mathbf{I_m \sin}\theta$.
The mean of the squares of the instantaneous values of current over one complete cycle is (even the value over half a cycle will do).

$$= \int_0^{2} \frac{{}^{2}d\theta}{2\pi - 0}$$

The square root of this value is $= \sqrt{\left(\int_0^{2\pi} \frac{i^2 d\theta}{2\pi}\right)}$

Hence, the r.m.s value of the alternating current is

$$I = \sqrt{\left(\int_0^{2\pi} \frac{i^2 d\theta}{2\pi}\right)} = \sqrt{\left(\frac{I_m{}^2}{2\pi} \int_0^{2\pi} \sin^2\theta d\theta\right)} \text{ (put } I = I_m \sin\theta)$$

Now, $\cos 2\theta = 1 - 2 \sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$I = \sqrt{\frac{i_m{}^2}{4\pi} \int_0^{2\pi} (1 - \cos 2\theta) d\theta} = \sqrt{\left(\frac{i_m{}^2}{4\pi} \left|\theta - \frac{\sin 2\theta}{2}\right|_0^{2\pi}\right)}$$

$$= \sqrt{\left(\frac{i_m{}^2}{4\pi} X 2\pi\right)} = \sqrt{\frac{i_m{}^2}{2}} \qquad \therefore I = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

Hence, we find that for a symmetrical sinusoidal current
RMS value of current = 0.707 X max. value of current.

Average heating effect is produced during 1 cycle $= I^2 R = \left(\frac{I_m{}^2}{\sqrt{2}}\right) R = \frac{1}{2} I_m{}^2 R$

RMS value of a complex wave:

In case of complex wave also either mid – ordinate method (when equation of the wave is not known) or analytical method (when equation of the wave is known)

Let a current equation be given by

I =12sin ωt +6 sin (3 ωt – 9/6) + 4∫ $(125\omega t + \frac{\pi}{3})$ flow through a resistor of R ohm.

Then, in the time period T second of the wave, the effect due to each component is as below

Fundamental – $(\frac{12}{\sqrt{2}})^2$ $RT\ watts$

3rd harmonic - $(\frac{6}{\sqrt{2}})^2$ $RT\ watts$

5th harmonic - $(\frac{4}{\sqrt{2}})^2$ $RT\ watts$

∴ Total heating effect of the complex wave, then equivalent heating effect is I^2RT

$$I^2RT = RT[(\frac{12}{\sqrt{2}})^2 + (\frac{6}{\sqrt{2}})^2 + (\frac{4}{\sqrt{2}})^2]$$

If I is the rms value of complex wave, then equivalent heating effect is I^2RT.

$$I^2RT = RT[(\frac{12}{\sqrt{2}})^2 + (\frac{6}{\sqrt{2}})^2 + (\frac{4}{\sqrt{2}})^2]$$

$$I = RT\sqrt{\left[\left(\frac{12}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2\right]} = 9.74\ A$$

If a direct current of 5 amp flowing of flowing in the circuit also, then the rms value would have been

$$\sqrt{\left[\left(\frac{12}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + 5^2\right]} = 10.93$$

∴ For a complex wave --- the rms value of a complex current wave is equal to the square root of the sine of the squares of the rms value of its individual components.

Calculate the rms value, form factor & of a periodic voltage having the following values for equal time intervals changing suddenly from one value to next – 0, 5, 10, 20, 50, 60, 50, 20, 10, 5, -10 -20 -50-20-10-5-0

$$V_{RMS} = \frac{0^2+5^2+10^2+20^2+50^2+60^2+50^2+20^2+10^2+5^2}{10}$$

$$= \sqrt{965} = 31V$$

$$V_{AV} = \frac{0+5+10+20+50+60+50+20+10+5}{10} = 23V$$

$$FF = \frac{\text{RMS value}}{\text{Average value}} = \frac{31}{23} = 1.35$$

Average value:

The average value I_a of an alternating current is expressed by that steady current which transforms across any circuit the same change as is transformed by that alternating current during the same time.

Hence, in their case, the average value is obtained by adding or integrating the instantaneous values of current over one half – cycle.

a) Mid – ordinate method

$$I_{AV} = \frac{i_1 + i_2 + i_3 + 4 \ldots + i_n}{n}$$

This method may be used for sinusoidal & non – sinusoidal waves.

b) Analytical method

The standard equation of an alternating current, $I = I_m \sin\theta$

$$I_{av} = {}_0^{\pi}\int \frac{i d\theta}{(\pi - 0)} = \frac{I_m}{\pi}\, {}_0^{\pi}\int \sin\,\theta\, d\theta$$

$$\frac{I_m}{\pi}[-\cos\theta]_0^{\pi} = \frac{I_m}{\pi}[+1-(-1)] = \frac{2I_m}{\pi} = \frac{I_m}{\pi/2}$$

$$\frac{\text{Twice the maximum current}}{\pi}$$

$$I_{av} = \frac{I_m}{1/2\pi} = 0.637\, I_m$$

[RMS value is always given than average value except in the case of a rectangular wave when both are equal]

Form factor:

$$K_f = \frac{rms\ value}{average\ value} = \frac{0.707\, I_m}{0.637\, I_m} = 1.1$$

Crest or peak or Amplitude factor:

It is defined as the ratio Ka $= \frac{\text{maximum value}}{\text{rms value}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} =$ 1.414(for sinusoidal a.c. only)

For sinusoidal alternating voltage also Ka $= \frac{E_m}{E_m/\sqrt{2}} =$ 1.414

Knowledge of this factor is of importance in dielectric insulation testing, because the dielectric stress to which the insulation is subjected, is proportional to the maximum or peak value of the applied voltage. The knowledge is also necessary when measuring iron losses, because the iron loss depends on the value of maximum flux.

Single phase AC circuits:

In dc circuits, voltage applied & current flowing are constant w.r.t time & to the solution to pure dc circuits can be analyzed simply by applying ohm`s law.

In ac circuits, voltage applied and currents flowing change from instant to instant.

If a single coil is rotated in a uniform magnetic field, the currents thus induces are called single phase currents.

A.C. Through pure Ohmic resistance only:

The circuit is shown in Fig Let the applied voltage be given by the equation.

$V = V_m \, sin\theta = V_m sin\omega t$

Let R = Ohmic resistance; I = instantaneous current.

Obviously, the applied voltage has to supply Ohmic voltage drop only. Hence for equilibrium

$V = iR$ (i)

Putting the value of V from above, we get $V_m sin\omega = iR; \; i = \frac{V_m}{R} sin\omega t$ (ii)

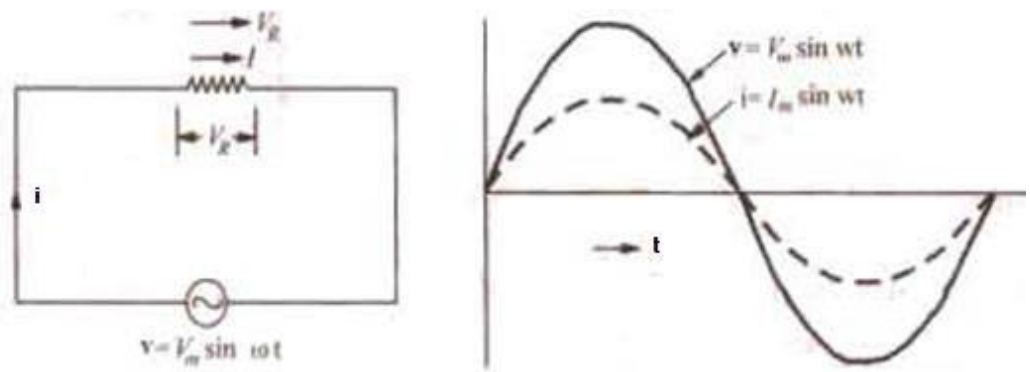
Current `i` is maximum when sinωt is unity

$\therefore I_m = V_m/R$

Hence, equation (ii) becomes,

$I = I_m \sin \omega t$

Comparing (i) And (ii), we find that the alternating voltage and current are in phase with each other as shown in fig. It is also shown vectorially by vectors V_R and I infig



Power. Instantaneous power, P = Vi = $\frac{VmIm}{2}$ - $\frac{VmIm}{2} cos\ 2\omega t$

Power consists of a constant part $\frac{VmIm}{2}$ and a fluctuating part $\frac{VmIm}{2} cos\ 2\omega t$ of frequency double that of voltage and current waves. For a complete cycle the average of $\frac{V_m I_m}{2} cos\ 2\omega t$ is zero

Hence, power for the whole cycle is

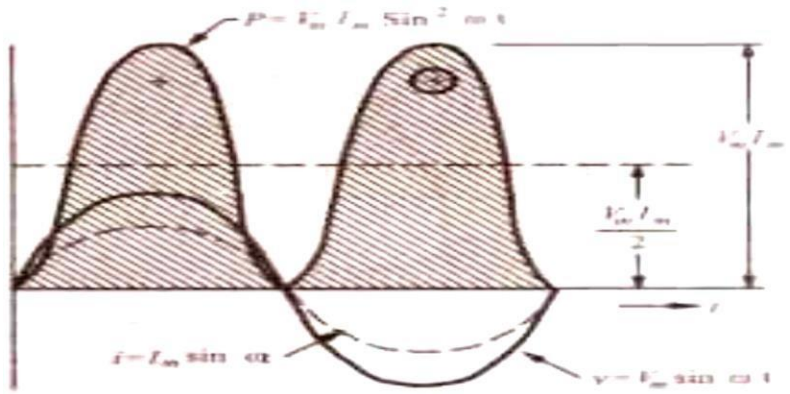
$$\mathbf{P} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} = \mathbf{X}\ \frac{I_m}{\sqrt{2}}$$

P = VI Watts

Where V = rms value of applied voltage.

I = rms value of the current.

It is seen from the fig that no part of the power cycle becomes negative at any time. In other words, in a purely resistive circuit, power is never zero. This is so because the instantaneous values of voltage and current are always either both positive and negative and hence the product is always positive.



A.C. Through Pure Inductance Alone:

Whenever an alternating voltage is applied to a purely inductive coil, a back e.m.f. is produced due to the self-inductance of the coil. As there is no Ohmic voltage drop, the applied voltage has to overcome this self – induced e.m.f. Only. So at every step

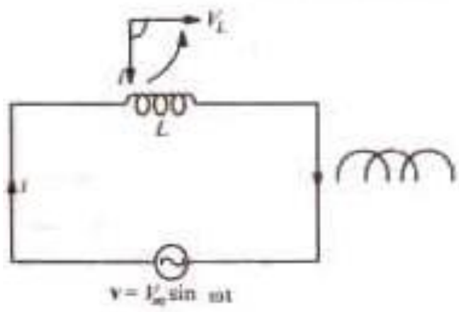
$$V = L\frac{di}{dt}$$

Now $\qquad V = V_m sin\omega t$

$$V_m sin\omega t\ = L\ \frac{di}{dt}\ \therefore di = \frac{V_m}{L} sin\omega t\ dt$$

Integrating both sides we get, I = $\frac{V_m}{L}\int\ sin\omega t\ dt$

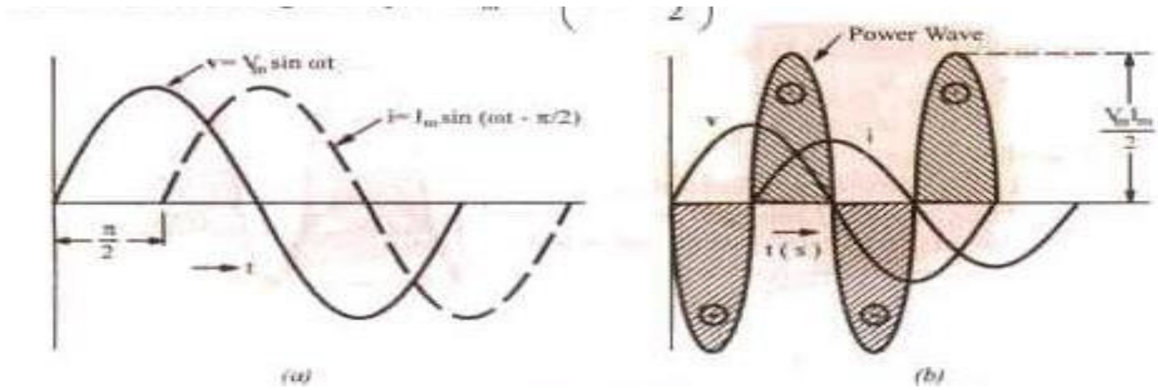
$$\frac{V}{\omega L}(-\cos \omega t)$$



$$\frac{V_m}{\omega L}(\sin \omega t - \frac{\pi}{2}) = \frac{V_m}{X_L}(\sin \omega t - \frac{\pi}{2})$$

Max value of I is $I_m = \frac{V_m}{\omega L}$ when $\sin(\omega - \frac{\pi}{2})$ is unity

Hence, the equation of the current becomes **I =** $I_m \sin(\omega t - \frac{\pi}{2})$



Clearly, the current lags behind the applied voltage by a quarter cycle (fig) or the phase deference between the two is $\frac{\pi}{2}$ with voltage leading. Vectors are shown in fig. where voltage has been taken along the reference axis. We have seen that

$$I_m = \frac{V}{\omega L} = \frac{V_m}{X_L}.$$

Here ωL plays the part of `resistance`. It is called the (inductive) reactance X_L of the coil and is given in ohms if L is in Henry and ω is in radians/second.

Now, $X_L = \omega L = 2\pi f l\ ohm$. It is seen that X_L depends directly on frequency of the voltage Higher the value of f, greater the reactance offered and vice-versa.

Power: Instantaneous power = : $V_i = V_m I_m \sin\omega t \sin\left(\omega t - \frac{\pi}{2}\right) = -\frac{V_m I_m}{2} \sin 2\omega t$

Power for whole cycle is $P = \frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \, dt = 0$

It is also clear from fig that th e average demand of power from the supply for a complex cycle is zero. Here again it is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of the instantaneous power is $\frac{VmIm}{2}$

A.C. Through pure capacitance alone :

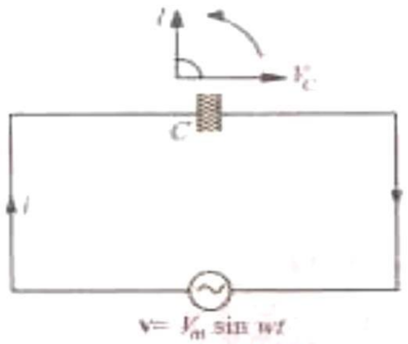
When an alternating voltage is applied to the plates of a capacitor, the capacitor is charged first in one direction and then in the opposite direction. When reference to fig.

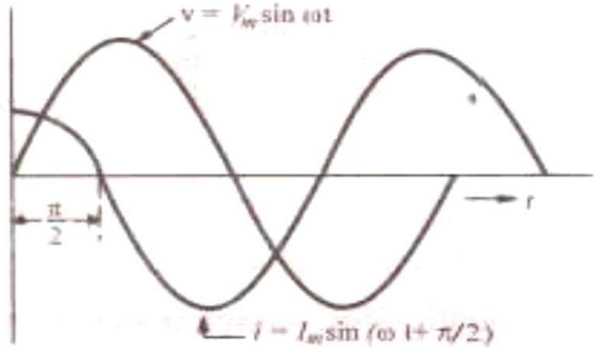
V = p.d. developed between plates at any instant.

q = Charge on plates at that instant.

Then q = cv (where C is the capacitance)

q= C V_m sin ωt. putting the value of v





Now, current I is given by the rate of flow of charge.

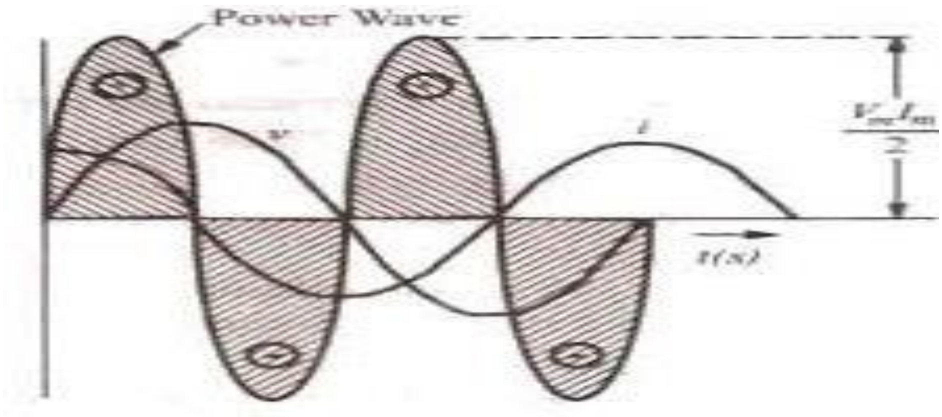
$\therefore$ i $= \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin \omega t) = \omega CV_m \cos\omega t$ or $i = \frac{V_m}{1/\omega c} \cos\omega t = \frac{V_m}{1/\omega c} \quad \sin\left(\omega t - \frac{\pi}{2}\right)$

Obviously, $I_m = \frac{V_m}{1/\omega c} = \frac{V_m}{\omega c} \therefore i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$

The denominator $X_c = 1/\omega C$ is known as capacitive reactance and is in ohms if C is in farad and ω in radian/second. It is seen that if the applied voltage is given by V = V_m sin ωt, then the current is given by I = I_m sin $(\omega t + \frac{\pi}{2})$.

Hence we find that the current in a pure capacitor leads its voltage by a quarter cycle as shown in fig. or phase difference between its voltage and current is $\frac{\pi}{2}$ with the current leading. Vector representation is given in fig. Note that V_c is taken along the reference axis.

Power Instantaneous power



$$P = V_m \sin\omega t I_m \sin(\omega t + 90^0)$$

$$= V_m I_m \sin \omega t \cos \omega t = \frac{1}{2} V_m I_m \sin 2\omega t$$

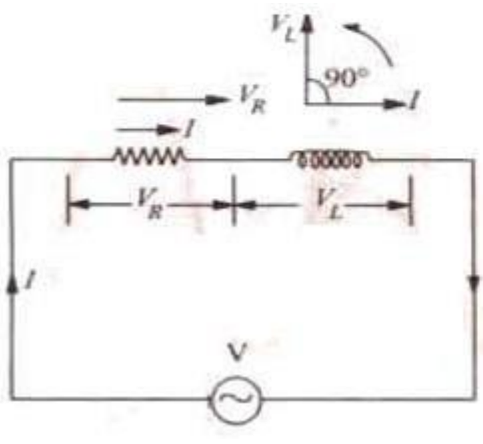
Power for the whole cycle

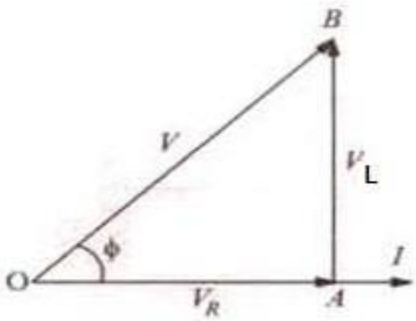
$$= \frac{1}{2} V_m I_m \int_0^{2\pi} \sin 2\omega t \, dt = 0$$

This fact is graphically illustrated in fig. we find that in a purely capacitive circuit. the average demand of power from supply is zero (as in a purely inductive circuit). Again, it is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of the instantaneous power is $\frac{VmIm}{2}$.

A.C. Through Resistance and inductance:

A pure resistance R and a pure inductive coil of inductance L are shown connected in series in fig.





Let V = r.m.s. value of the applied Voltage, I = r.m.s. value of the resultant current V_R = IR – Voltage drop across R (in phase with I), $V_L = I.X_L$ – voltage drop across coil (ahead of I by 90^0)

These voltage drops are shown in voltage triangle OAB in fig. Vector OA represents Ohmic drop V_R and AB represents inductive drop V_L. The applied V is the vector sum of the two i.e. OB

$$\therefore V = \sqrt{\left(V_R{}^2 + V_L{}^2\right)} = \sqrt{[(IR^2 + (I.X_L)^2)]} = I\sqrt{R^2 + X_L{}^2}, \frac{V}{\sqrt{(R)^2 + X_L{}^2}} = I$$

The quantity $\sqrt{R^2 + X_L{}^2}$, is known as the impedance (Z) of the circuit. As seen from the Impedance triangle ABC (fig,) $Z^2 = R^2 + X_L{}^2$

i.e $(\text{impedance})^2 = (\text{resistance})^2 + (\text{Reactance})^2$

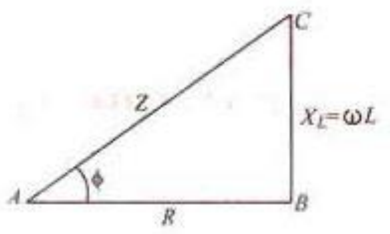
From fig. it is clear that the applied voltage V leads the current I by an angle Φ such that

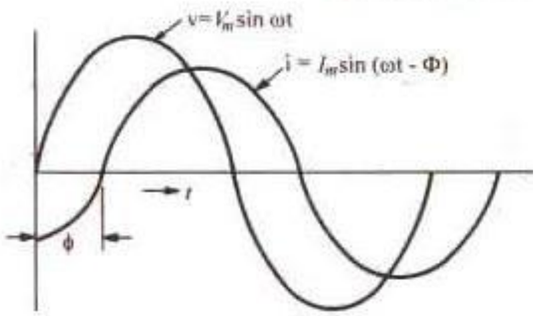
$$\tan\Phi = \frac{V_L}{V_R} = \frac{I\ .X_L}{I.R} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{reactance}{reactance} \therefore \emptyset = tan^{-1}\frac{X_L}{R}$$

The same fact is illustrated graphically in fig.

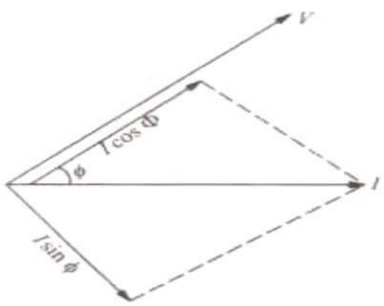
In other words, current I lags behind the applied voltage V by an angle ∅. Hence, if applied voltage is given by $v = V_m \sin \omega t$, then current equation is

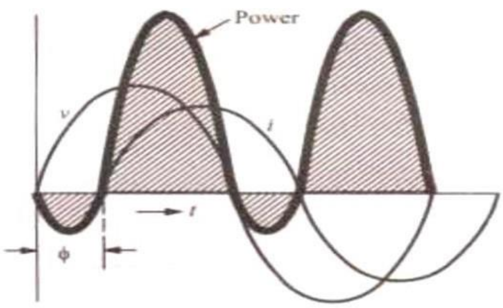
$$i = I_m \sin(\omega t - \emptyset) \text{ where } I_m = V_m/Z$$





IIn fig. I has been resolved in to its two mutually perpendicular components, I cos Φ along the applied voltage V and I sin Φ in quadrature (i.e. perpendicular) with V.





The mean power consumed by the circuit is given by the product of V and that component of the current I which is in phase with V

So P = V X I cos Φ = r.m.s. voltage X r.m.s. current X cos Φ

The term cos Φ is called the power factor of the circuit

Remember that in an a.c. circuit, the product of r.m.s. amperes gives volt ampere (VA) and not true power in watts. True power (W) = volt amperes (VA) power factor.

Or Watts = VA cos Φ^0 Or

It should be noted that power consumed is due to Ohmic resistance only because pure inductance does not consume any power.

Now P = VI cos Φ = VI X (R/Z) = V/Z X IR = I^2R (∵ cos Φ = R/Z) or P = I^2R watt.

Graphical representation of the power consumed is shown in fig.

Let us calculate power in terms of instantaneous values.

Instantaneous power is = vi = $v_m \sin \omega t$ X $I_m \sin(\omega t - \Phi)$ = $V_m I_m \sin \omega t \sin(\omega t - \Phi)$

$$\frac{1}{2} V_m I_m [\cos\Phi - \cos(2\omega t - \Phi)]$$

Obviously this consists of two parts

A constant part $\frac{1}{2} V_m I_m$ of which contributes to real power

A pulsating component $\frac{1}{2} V_m I_m \cos(2\omega t - \Phi)$ which has a frequency twice that of the voltage and current. It does not contribute to actual power since its average value over a complete cycle is zero.

Hence, average power consumed $\frac{1}{2} V_m I_m \cos\Phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\Phi = VI\cos\Phi$ where V and I represent rms values.

Symbolic notation

$$Z = R + jX_L$$

Impedance vector has numerical value of $\sqrt{(R^2 + X_L^2)}$.

Its phase angle with the reference axis is $\phi = \tan^{-1}(X_L/R)$

It may also be expressed in the polar form as $\mathbf{Z} = \mathbf{Z} \angle \phi°$

i) Assuming

$$\mathbf{V} = V \angle 0°; \mathbf{I} = \frac{V}{Z} = \frac{V \angle 0°}{Z \angle \phi°} = \frac{V}{Z} \angle -\phi°$$

It shows that current vector is lagging behind the voltage vector by

The numerical value of current is

ii) However, If we assume that

$$\mathbf{I} = I\angle 0, \text{ then}$$
$$\mathbf{V} = \mathbf{IZ} = I\angle 0^\circ \times Z\angle \phi^\circ$$
$$= IZ\angle \phi^\circ$$

It shows that voltage vector is ϕ° a head of current vector is ccw direction as shown in fig.

Power factor: it may be defined as

i) Cosine of the angle of lead or lag

ii) The ratio R/Z = $\dfrac{Resistance}{Impedance}$

iii) The ratio = $\dfrac{\text{True power}}{\text{Apperent power}} = \dfrac{\text{Watts}}{\text{Volt−amp}} = \dfrac{\text{W}}{\text{VA}}$

Active and reactive components of circuit current I :

Active component is that which is in phase with the applied voltage V i.e $I\cos\Phi$. It is also known as 'wattful' component.

Reactive component is that which quadrature is with V i.e. $I\sin\Phi$ it is also known as 'watt less' or 'idle' component.

It should be noted that the product of volts and amperes in an a.c. circuit gives volt-amperes (VA). Out of this, the actual power is VA $\cos\Phi$ = W and reactive power is $VA\sin\phi$. expressing the values in KVA, we find that it has two regular components :

(1) Active component which is obtained by multiplying KVA by and this gives power in KW.

(2) The reactive component known as reactive KVA and is obtained by multiplying KVA by. $\sin\phi$. It is written as KVAR (kilovar). The following relations can be easily deduced.

$$\text{kVA} = \sqrt{\text{kW}^2 + \text{kVAR}^2} \text{ ; kW} = \text{kVa}\cos\phi \text{ and kVAR} = \text{kVA}\sin\phi$$

These relationships can be easily understood by referring to the KVA triangle of fig.13.10. where it should be noted that lagging KVAR has been taken as negative.

For example, suppose a circuit draws a current of 1000A at a voltage of 20,000 V and has a power factor of 0.8. Then input = 1,000 × 20,000/1000 = 20,000 kVA; $\cos\phi = 0.8$; $\sin\phi = 0.6$

Hence kW = 20,000 × 0.8 = 16,000 ; kVAR = 20,000 × 0.6 = 12,000

Obviously, $\sqrt{16000^2 + 12000^2} = 20,000$ *i.e.* $\text{kVA} = \sqrt{\text{kW}^2 + \text{kVAR}^2}$

ACTIVE, REACTIVE AND APPARENT POWER

Let a series circuit draw a current of when an alternating voltage of r.m.s value V is applied to it. suppose that current lags behind the applied voltage by Φ. The three powers drawn by the circuit are as under:

i) **Apparent power(s):** It is given by the product of rms values of applied Voltage and circuit current.

$S = VI = (IZ).I = I^2Z$ volt-amperes (VA)

ii) **Active power (P or W):** It is the power which is actually dissipated in the circuit resistance. $P = I^2R = VI\cos\Phi$ watts

iii) **Reactive power (Q) :** It is t he power developed in the inductive reactance of the circuit.

$Q = I^2X_L = I^2.Z\sin\Phi = I\,.\,(IZ).\sin\Phi = VI\sin\Phi$ volt-amp reactive (VAR)

These three powers are shown in the power triangle of fig. from where it can be seen that

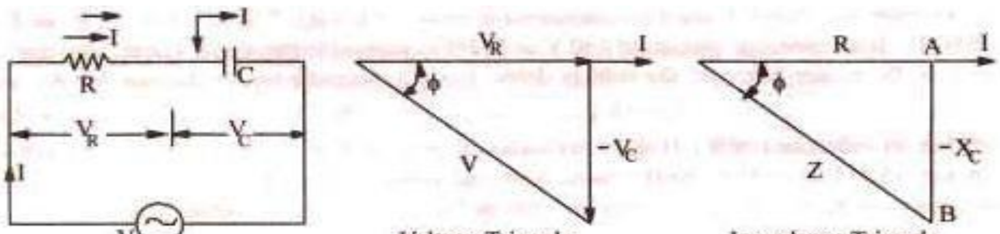
$S^2 = P^2 + Q^2$ or $\sqrt{P^2 + Q^2}$.

A.C. Through Resistance and capacitance:

This circuit is shown in fig. here $V_R = IR$ = drop across R in phase with I.

$V_C = IX_C$ = drop across capacitor –lagging I by $\pi/2$

As capacitive reactance X_c is taken negative, V_c is shown along negative direction if Y- axis in the voltage triangle



Voltage Triangle Impedance Triangle

(a) (b) (c)

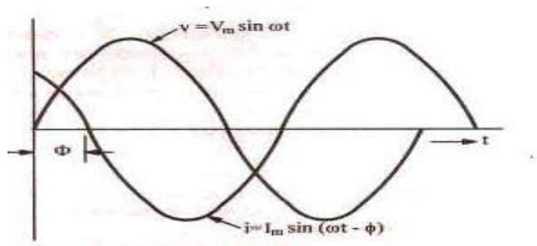
Now $V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2} = I\sqrt{R^2 + X_C^2}$ or $I = \dfrac{V}{\sqrt{R^2 + X_C^2}} = \dfrac{V}{Z}$

The denominator is called the impedance of the circuit. So $Z = \sqrt{R^2 + X_C^2}$

Impedance triangle is shown in fig.

From fig. (b) it is found that I leads V by angle ϕ such that $\phi = -X_c/R$

Hence, it means that if the equation of the applied alternating voltage is v = V_m sinωt, the equation of the resultant current in the T-C circuit is I = I_m sin (ωt + Φ) so that current leads the applied voltage by an angle ϕ. This fact is shown graphically in fig.



Example: An A.C. voltage (80+j 600 volts is applied to a circuit and the current flowing is (-4+j 10) amperes. Find (i) inpedance of the circuit (ii) power consumed and (iii) phase angle.

Solution:

Solution. $\mathbf{V} = (80 + j\,60) = 100 \angle 36.9°$;

V = $\mathbf{I} = -4 + j\,10 = 10.77 \angle \tan^{-1}(-2.5) = 10.77 \angle (180° - 68.2°) = 10.77 \angle 111.8°$

(*i*) $\mathbf{Z} = \mathbf{V/I} = 100 \angle 36.9°/10.77\angle 111.8°$

$= 9.28 \angle -74.9°$

$= 9.28 (\cos 74.9° - j \sin 74.9°) = 2.42 - j\,8.96\ \Omega$

Hence $R = 2.42\ \Omega$ and $X_C = 8.96\ \Omega$ capacitive

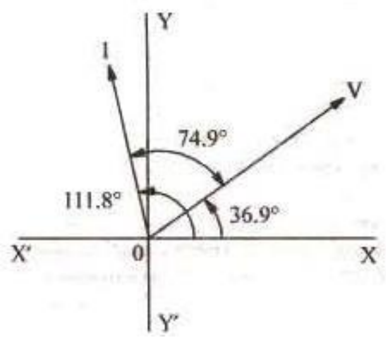
(*ii*) $P = I^2 R = 10.77^2 \times 2.42 = \mathbf{2.81\ W}$

Hence

$R = 2.42\ \Omega$ and $X_C = 8.96\ \Omega$ capacitive

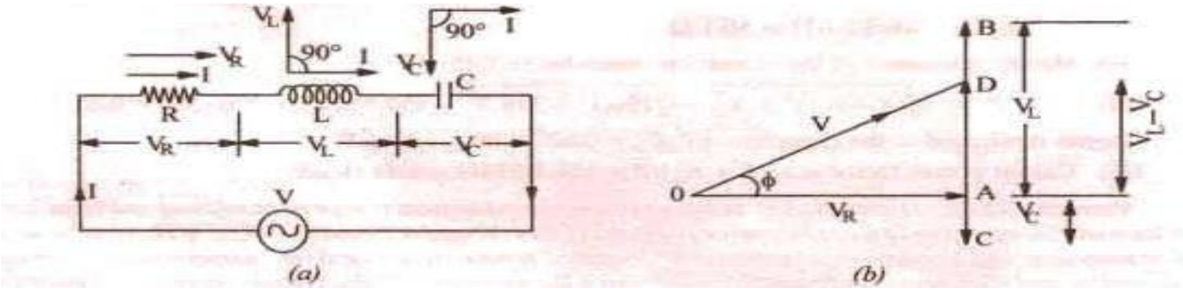
(*ii*) $P = I^2 R = 10.77^2 \times 2.42 = \mathbf{2.81\ W}$

(iii) Phase angle between voltage and current = 74.9_0 with current leading as shown.



Resistance, Inductance and Capacitance in series:

The three are shown in fig. (a) Joined in series across an a.c. supply of r.m.s. voltage V



Let V_R = IR = Voltage drop across R --------in phase with I

$V_L = I.X_L$ = Voltage drop across L ---- Leading I by $\pi/2$

$V_C = IX_C$ = = Voltage drop across C ---- Lagging I by $\pi/2$

$$\therefore \quad OD = \sqrt{OA^2 + AD^2} \quad \text{or} \quad V = \sqrt{(IR)^2 + (IX_1 - IX_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\text{or} \quad I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + X^2}} = \frac{V}{Z}$$

Then the term $\sqrt{R^2 + (X_L - X_C)^2}$ is known as the impedance of the circuit.
Obviously, $(\text{impedance})^2 = (\text{resistance})^2 + (\text{net reactance})^2$

$$\text{or} \quad Z^2 = R^2 + (X_l - X_c)^2 = R^2 + X^2$$

Where X is the net reactance (fig)

Phase angel Φ is given by $\tan\phi = (X_L - X_C)/R = X/R =$ net reactance /resistance Power factor is

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + X^2}}$$

Hence, it is seen that if the equation of the applied voltage is $v = V_m \sin \omega t$, then equation of the resulting current in an R-L-C circuit is given by

$i = I_m \sin(\omega t \pm \phi)$ $X_C > X_L$.

The positive sign is to be used when current lags i,e, $X_L > X_C$.

The negative sign is to be used when current lags i.e when
In general, the current lags or leads the supply voltage by an angle Φ

$\tan\phi = X/R$

such that

Using symbolic notation, we have

$\mathbf{Z} = R + j(X_L - X_C)$ Numerical value of impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Its phase is $\Phi = \tan^{-1}[X_L - X_C/R]$

$Z = Z\angle\tan^{-1}[(X_L - X_C)/R] = Z\angle\tan^{-1}(X/R)$

If $\mathbf{V} = V\angle 0$, then, $\mathbf{I} = \mathbf{V}/\mathbf{Z}$

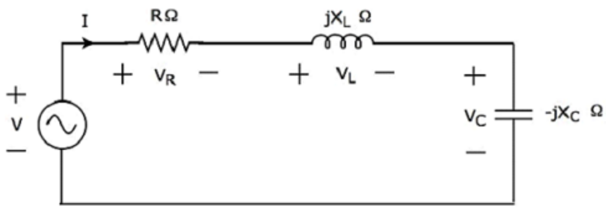
RESONANCE:

Resonance occurs in electric circuits due to the presence of energy storing elements like inductor and capacitor. It is the fundamental concept based on which, the radio and TV receivers are designed in such a way that they should be able to select only the desired station frequency.

There are two types of resonances, namely series resonance and parallel resonance. These are classified based on the network elements that are connected in series or parallel. In this chapter, let us discuss about series resonance.

Series Resonance Circuit Diagram:

If the resonance occurs in series RLC circuit, then it is called as Series Resonance. Consider the following series RLC circuit, which is represented in phasor domain.



Here, the passive elements such as resistor, inductor and capacitor are connected in series. This entire combination is in series with the input sinusoidal voltage source.

Apply KVL around the loop.

$$V - V_R - V_L - V_C = 0$$

$$\boxed{\Rightarrow V - IR - I(jX_L) - I(-jX_C) = 0}$$

$$\Rightarrow V = IR + I(jX_L) + I(-jX_C)$$

$$\Rightarrow V = I[R + j(X_L - X_C)]$$ **Equation 1**

The above equation is in the form of ***V = IZ***.

Therefore, the **impedance Z** of series RLC circuit will be

$$Z = R + j(X_L - X_C)$$

Parameters & Electrical Quantities at Resonance

Now, let us derive the values of parameters and electrical quantities at resonance of series RLC circuit one by one.

Resonant Frequency:

The frequency at which resonance occurs is called as resonant frequency f_r. In series RLC circuit resonance occurs, when the imaginary term of impedance Z is zero, i.e., the value of $X_L - X_C$ should be equal to zero.

$$\Rightarrow X_L = X_C$$

Substitute $X_L = 2\pi f L$ and $X_C = \frac{1}{2\pi f C}$ in the above equation.

$$2\pi f L = \frac{1}{2\pi f C}$$

$$\Rightarrow f^2 = \frac{1}{(2\pi)^2 LC}$$

$$\Rightarrow f = \frac{1}{(2\pi)\sqrt{LC}}$$

Where, L is the inductance of an inductor and C is the capacitance of a capacitor.

The resonant frequency f_r of series RLC circuit depends only on the inductance L and capacitance C. But, it is independent of resistance R.

Impedance:

We got the impedance Z of series RLC circuit as

$$Z = R + j(X_L - X_C)$$

Substitute $X_L = X_C$ in the above equation.

$$Z = R + j(X_C - X_C)$$

$$\Rightarrow Z = R + j(0)$$

$$\Rightarrow Z = R$$

At resonance, the impedance Z of series RLC circuit is equal to the value of resistance R, i.e.,

$Z = R$.

Current flowing through the Circuit:

$X_L - X_c = 0$ Substitute in equation 1

$V = I(R+J0)$

$I = V/R$

At resonance, the impedance of series RLC circuit reaches to minimum value. Hence, the maximum current flows through this circuit at resonance.

At resonance condition :

1.minimium impedance

2.maximum current

3.power factor is unity

4.circuit will act as purely resistive circuit.