1. Linear Differential Eqn's.

$$\frac{dy}{dx} + P(x)\,y = Q(x).$$

dep

Ind

$$\boxed{IF = e^{\int P(x)\,dx}}$$

Working Rule.

① $\frac{dy}{dx} + P(x)y = Q(x)$

② $IF = e^{\int P(x)\,dx}$

③ $y \times IF = \int (Q(x) \times IF)\,dx + c.$

Solve $x\frac{dy}{dx} + y = \log x$ — ①.

Sol eqn ① divide by x.

① ⇒ $\frac{\cancel{x}}{\cancel{x}}\frac{dy}{dx} + \frac{y}{x} = \frac{\log x}{x}.$

⇒ $\frac{dy}{dx} + \left(\frac{1}{x}\right)y = \frac{\log x}{x}$ — ②

∴ eqn ② is of form

(2)

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$P(x) = \frac{1}{x}, \quad Q(x) = \frac{\log x}{x}, \quad IF = e^{\int P(x)\,d}$$

$$IF = e^{\int \frac{1}{x} dx}$$

$$= e^{\log x}$$

$$= x.$$

$$\boxed{\therefore IF = x}.$$

Soln of LDE is

$$\Rightarrow y \times (IF) = \int [Q(x) \times IF]\,dx + C.$$

$$= y \times (x) = \int \left(\frac{\log x}{\cancel{x}} \times \cancel{x}\right) dx + C.$$

$$= xy = \int (1 \cdot \log x)\,dx + C$$

$$= xy = x \log x - x.$$

$$\boxed{= xy = x(\log x - 1) + C.}$$

LIATE.

$$\int f(x)\,g(x)\,dx.$$

$$= f(x) \int g(x)\,dx - \int \left[\frac{d}{dx} f(x) \int g(x)\,dx\right] d$$

1st func Integral of 2nd − [Int g

der of first × Int of 2nd] d

$$f(x) = \log x.$$

$$\int g(x)\,dx = \int 1\,dx$$

$$\frac{d}{dx}(f(x)) = \frac{1}{x}.$$

$$\int \int g(x)\,dx = x.$$

Solve $\frac{dy}{dx} + \frac{y}{x\log x} = \frac{\sin 2x}{\log x}$ — ①.

$\Rightarrow$ eqn ① is of the form

$$\frac{dy}{dx} + P(x)y = Q(x). \Rightarrow \frac{dy}{dx} + \left(\frac{1}{x\cdot\log x}\right)y = \frac{\sin 2x}{\log x}.$$

$$P(x) = \frac{1}{x\log x}, \quad Q(x) = \frac{\sin 2x}{\log x}.$$

$$IF = \cancel{\int P(x)\,dx}\ e^{\int P(x)\,dx}.$$

$$e^{\int\left(\frac{1}{x\log x}\right)dx}$$

$$e^{\int\frac{dx}{x\log x}}.$$

$$e^{\int\frac{\frac{1}{x}dx}{\log x}}.$$

$$e^{\log(\log x)}$$

$$\int\frac{f'(x)}{f(x)} = \log|f(x)|.$$

$$\boxed{IF = \log x.}$$

$$\int \sin 2x = \frac{-\cos 2x}{2}.$$

Soln $y \times IF = \int(Q(x).\,IF)\,dx + C.$

$$y\log x = \int\left(\frac{\sin 2x}{\log x}.\log x\right)dx + C.$$

$$y\log x = \int(\sin x)\,dx + C.$$

$$y\log x = \frac{-\cos 2x}{2} + C.$$

(4)

Solve $x^2 \frac{dy}{dx} = 3x^2 - 2xy + 1$.

$\Rightarrow$ $\boxed{\frac{dy}{dx} + P(x)\,y = Q(x)}$ is the LDE.

$\Rightarrow$ divide by x^2 on B.S.

$$\frac{x^2}{x^2}\frac{dy}{dx} = \frac{3x^2}{x^2} - \frac{2xy}{x^2} + \frac{1}{x^2}.$$

$$\frac{dy}{dx} = 3 - \frac{2y}{x} + \frac{1}{x^2}.$$

$$\frac{dy}{dx} + \left(\frac{2}{x}\right)y = \left(3 + \frac{1}{x^2}\right).$$

$\int 1\,dx = x$

$\frac{x^{n+1}}{n+1}$

$\Rightarrow$ $P(x) = \frac{2}{x}$, $Q(x) = 3 + \frac{1}{x^2}$.

$\Rightarrow$ $IF = e^{\int P(x)\,dx}$

$= e^{\int \frac{2}{x}\,dx}$

$= e^{2\log x}$

$= e^{\log x^2}$

$IF = x^2$.

$\Rightarrow$ $y \times IF = \int [Q(x) \times IF]\,dx + c$.

$y x^2 = \int \left[\left(3 + \frac{1}{x^2}\right) \times x^2\right] dx + c$.

$y x^2 = \int 3x^2\,dx + c$.

$y x^2 = \int 3x^2\,dx + \int 1\,dx + c$.

$y x^2 = \frac{3x^3}{3} + x + c$.

$y x^2 = x^3 + x + c$.

$\boxed{y x^2 = x(x^2+1) + c}$

is the soln of given LDE.

olve $dr + (2r\cot\theta + \sin 2\theta)d\theta = 0$.

$$\frac{dr}{d\theta} + 2r\cot\theta + \sin 2\theta = 0$$

$$\frac{dy}{dx} + P(x)\,y = Q(x)$$

$$\frac{dr}{d\theta} + 2r\cot\theta = -\sin 2\theta$$

$$\frac{dr}{d\theta} + (2\cot\theta)r + \sin 2\theta = 0 \quad -(1)$$

eqn (1) is of the form

$x = \theta$
$y = r$

$$\frac{dy}{dx} + P(x)\,y = Q(x)$$

$\Rightarrow$ $P(\theta) = 2\cot\theta$, $Q(\theta) = \sin 2\theta$.

$$\Rightarrow \; IF = e^{\int P(x)dx}$$
$$= e^{\int P(\theta)d\theta}$$
$$= e^{\int (2\cot\theta)d\theta}$$
$$= e^{2\int \cot\theta\, d\theta}$$
$$= e^{2\int \frac{\cos\theta}{\sin\theta}d\theta}$$

$$\int \frac{f'(x)}{f(x)} = \log|f(x)|$$

$$= e^{2\log|\sin\theta| + c}$$
$$= e^{\log(\sin\theta)^2}$$

$$\boxed{IF = \sin^2\theta}$$

(2) (6)

~~sdn.~~ $y \times \cancel{R(x)}\, IF = \int [Q(x) \times IF]\, dx + c.$

$r \times IF = \int [Q(\theta) \times IF]\, d\theta + c.$

$\boxed{\sin 2\theta = 2 \ldots \cos}$

$r \sin^2\theta = \int [\sin 2\theta \times \sin^2\theta]\, d\theta + c.$

sdn $r \sin^2\theta = \int (2 \sin\theta \cos\theta \times \sin^2\theta)\, d\theta + c.$

$\boxed{\begin{array}{l} \sin\theta = t. \\ \cos\theta\, d\theta = dt \end{array}}$

$r \sin^2\theta = 2 \int (\sin^3\theta \cos\theta)\, d\theta + c.$

$r \sin^2\theta = 2 \int t^3\, dt + c.$

$\boxed{t^n = \frac{t^{n+1}}{n+1}}$

$r \sin^2\theta = \cancel{2}\, \frac{t^4}{4/2} + c.$

$r \sin^2\theta = \frac{t^4}{2} + c.$

LCM.

$\boxed{2 r \sin - t^4 = 2c}$

Solve $(1+y^2)dx = (\tan^{-1}y - x)dy$.

$$(1+y^2)\frac{dx}{dy} = \tan^{-1}y - x \quad \text{---} \ ①$$

$① \div 1+y^2$.

$$\frac{\cancel{1+y^2}}{\cancel{1+y^2}}\frac{dx}{dy} = \left(\frac{\tan^{-1}y - x}{1+y^2}\right)$$

$$\frac{dx}{dy} = \frac{\tan^{-1}y}{1+y^2} - \frac{x}{1+y^2}.$$

$$\boxed{\frac{dx}{dy} + P(y)x = Q(y)}$$

$$\frac{dx}{dy} + \left(\frac{1}{1+y^2}\right)x = \frac{\tan^{-1}y}{1+y^2} \quad ②$$

can ② $\Rightarrow$

$$P(y) = \frac{1}{1+y^2}, \quad Q = \frac{\tan^{-1}y}{1+y^2}$$

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$$IF = e^{\int P(y)dy}$$

$$= e^{\int \frac{1}{1+y^2} dy}$$

$$\boxed{\int \frac{1}{1+y^2} =}$$

$$e^{\tan^{-1} y}$$

$$IF = e^{\tan^{-1} y}$$

Soln $x \times IF = \int (Q(y) \times IF)\, dy + c$

$$x \times e^{\tan^{-1} y} = \int \left(\frac{\tan^{-1} y}{1+y^2} \times e^{\tan^{-1} y} \right) dy + c$$

Submitt

$\tan^{-1} y = t$

$\frac{1}{1+y^2} dy = dt$

$$x e^{t} = \int [t e^t \, dt] + c$$

ILATE

$$\boxed{\int f(x) g(x) dx = f(x) \int g(x) dx - \int \left[\frac{d}{dx} f(x) \int g(x) dx \right] dx}$$

$f(x) = t$

$g(x) = e^t$

$$= x e^t = t \int e^t dt - \int \left[\frac{d}{dx} t \cdot \int e^t dt \right] dt + c$$

$$1 \qquad e^t$$

$$x e^t = t e^t - \int e^t dt + c$$

$$x e^t = t e^t - e^t + c$$

$$x e^t = e^t (t-1) + c$$

$$\boxed{x e^{\tan^{-1} y} = e^{\tan^{-1} y} \{\tan^{-1} y - 1\} + c}$$

$$\sin 2x \frac{dy}{dx} - y = \tan x$$

$$\boxed{\frac{dy}{dx} + P(x)y = Q(x)}$$

$$\frac{\cancel{\sin 2x}}{\cancel{\sin 2x}} \frac{dy}{dx} + \frac{1(y)}{\sin 2x} = \frac{\tan x}{\sin 2x}$$

$$\frac{dy}{dx} \cancel{+} \left(\frac{1}{\sin 2x}\right) y = \frac{\tan x}{\sin 2x} \quad -①$$

$$\frac{dy}{dx} - \left(\frac{1}{\sin 2x}\right) y = \frac{1}{2\cos^2 x}$$

$$\frac{\sin x}{\cos x} \Big/ \frac{\sin x}{1}$$

$$\frac{\cancel{\sin x}}{\cancel{\sin x}} \frac{1}{\cos x}$$

$=$

$$\boxed{\frac{\tan x}{\sin 2x} = \frac{1}{2\cos^2 x}}$$

$$\frac{dy}{dx} (-\operatorname{cosec} 2x) y = \frac{1}{2\cos^2 x}$$

$$P(x) = -\operatorname{cosec} 2x, \quad Q(x) = \frac{1}{2\cos^2 x}$$

$$IF = e^{\int P(x)\,dx}$$

$$= e^{\int(-\operatorname{cosec} 2x)\,dx}$$

$$= e^{-\int \operatorname{cosec} 2x\,dx}$$

$$= e^{\frac{1}{2}[\operatorname{cosec} 2x + \cot 2x]}$$

$$= e^{(\operatorname{cosec} 2x + \cot 2x)\frac{1}{2}}$$

$$\boxed{\int \operatorname{cosec} x = -\ln|\operatorname{cosec} x + \cot x|}$$

$$\boxed{\int \operatorname{cosec} 2x = \frac{-1}{2}|\operatorname{cosec} 2x + \cot 2x|}$$

$$IF = (\operatorname{cosec} 2x + \cot 2x)^{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{\operatorname{cosec} 2x + \cot 2x}}$$

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta}$$

$$\boxed{\begin{aligned}&\operatorname{cosec} 2x + \cot 2x \\ &= \tan x\end{aligned}}$$

$$= \frac{1 - \cos 2\theta}{\sin 2\theta}$$

$$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\boxed{= \frac{1}{\sqrt{\tan x}}}$$

$$y \times IF = \int [Q(x) \times IF]\, dx + c$$

$$y \times \frac{1}{\sqrt{\tan x}} = \int \left(\frac{\tan x}{\sin 2x} \cdot \frac{1}{\sqrt{\tan x}}\right) dx + c$$

$$= \frac{\sqrt{\cancel{\tan x}} \cdot \sqrt{\tan x}}{\sin 2x \sqrt{\cancel{\tan x}}}\, dx + c$$

$$y \times \frac{1}{\sqrt{\tan x}} = \int \sec^2 x \sqrt{\tan x}\, dx + c$$

$$\frac{y}{\sqrt{\tan x}} = \sqrt{\tan x} + c$$

$$\boxed{\int \sec^2 x \sqrt{\tan x}\, dx = \sqrt{\tan x}}$$

Bernoullis Diff.

An eqn of form $\frac{dy}{dx} + P(x)\,y = Q(x)\,y^n$ — (1).

case(1) ÷ If $\boxed{n=1}$ eqn (1) can be written as

$$\frac{dy}{dx} + [P(x) - Q(x)]\,y = 0 \quad \text{— (2)}$$

use variable seperation method

$$\int \frac{dy}{y} + \int (P-Q)\,dx = c$$

case ii) If $n \neq 1$ (1) ÷ with y^n

$$y^{-n}\frac{dy}{dx} + \frac{P(x)\,y}{y^n} = \frac{Q(x)\,y^n}{y^n}$$

$$y^{-n}\frac{dy}{dx} + P(x)\,y^{1-n} = Q(x) \quad \text{— (3)}$$

let $y^{1-n} = u$.

$$(1-n)\,y^{-n}\frac{dy}{dx} = \frac{du}{dx}$$

$$y^{-n}\frac{dy}{dx} = \frac{1}{1-n}\frac{du}{dx} \quad \text{— (4)}$$

$\therefore$ ③ & ④.

③ $\Rightarrow \frac{1}{1-n}\frac{du}{dx} + P(x)\,u = Q(x)$.

i.e $\frac{du}{dx} + (1-n)\,P(x)u = (1-n)\,Q(x)$.

This is the linear eqn of first order u

Solve $x\frac{dy}{dx} + y = x^3 y^6$.

$$\boxed{\frac{dy}{dx} + P(x)y = Q.(x).\,y^{n}}$$

div by x.

$$\frac{\cancel{x}}{\cancel{x}}\frac{dy}{dx} + \frac{y}{x} = \frac{x^3\,y^6}{x}$$

$$\frac{dy}{dx} + \left(\frac{1}{x}\right)y = x^2 . y^6 \quad \text{—①}$$

div by y^6.

$$\frac{1}{y^6}\frac{dy}{dx} + \frac{1}{x.y^6}\,y = x^2$$

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$$\boxed{\frac{1}{y^6}\frac{dy}{dx}} + \frac{1}{x}\boxed{\frac{1}{y^5}} = x^2 \quad -(2)$$

put = $\boxed{\frac{1}{y^5} = u}$ $-(3)$

$$u = y^{-5}$$

$$\frac{du}{dx} = -5y^{-6}\frac{dy}{dx}$$

$$\frac{du}{dx} = -\frac{5}{y^6}\frac{dy}{dx}$$

$$\boxed{-\frac{1}{5}\frac{du}{dx} = \frac{1}{y^6}\frac{dy}{dx}} \quad -(4)$$

put (3) & (4) in (2)

$\times 5$

(2) $\Rightarrow$ $\boxed{-\frac{1}{5}\frac{du}{dx} + \frac{1}{x}u = x^2}$ $-(5)$

$$\Rightarrow -\frac{du}{dx} - \frac{5}{x}u = 5x^2$$

$$\boxed{\frac{du}{dx} - \frac{5}{x}u = -5x^2} \quad -(6)$$

Eqn ⑥ is of form.

$$\boxed{P(x) = -\frac{5}{x}}$$

$$\frac{dy}{dx} + P(x)y = Q(x).$$

$Q(x) = -5$

$$IF. = e^{\int P(x)dx}.$$

$$= e^{-\int \frac{5}{x} dx}$$

$$= e^{-5\log x}.$$

$$= e^{\log x^{-5}}.$$

$$IF. = x^{-5}.$$

$$\boxed{IF. = -\frac{1}{x^5}}.$$

Sol. $u \times IF = \int [Q(x) \times IF] dx + c.$

$$\Rightarrow u \times \left(\frac{-1}{x^5}\right) = \int \left[-5x^2 \cdot \frac{1}{x^5}\right] dx + c$$

$$\frac{-u}{x^5} = -5\int \frac{1}{x^3} dx + c.$$

$$-\frac{u}{x^5} = -5\int x^{-3} dx + c.$$

$$\frac{-u}{x^5} = \frac{5}{2x^2} + c. \Rightarrow \boxed{\frac{-1}{y^5 x^5} = \frac{5}{2x^2} + c.}$$

Newtons Laws of cooling :- The rate of change of temp of body is prop to diff of the temp of body & that of surrounding medi. θ_0.

$$\frac{d\theta}{dt} \propto \theta - \theta_0.$$

$$\Rightarrow \frac{d\theta}{dt} = -k(\theta - \theta_0).$$

$$\int \frac{d\theta}{(\theta - \theta_0)} = \int -k\,dt$$

$$\log(\theta - \theta_0) = -kt + \log c.$$

$$\log(\theta - \theta_0) - \log c = -kt$$

$$= \log\frac{(\theta - \theta_0)}{c} = -kt$$

$$\frac{\theta - \theta_0}{c} = e^{-kt}$$

$$\theta - \theta_0 = ce^{-kt}.$$

$$\boxed{\theta = \theta_0 + ce^{-kt}}$$

A body kept in air with temp $25°C$ cools from $140°$ to $80°$ in 20 min. Find when the body cools down to $35°C$.

Sol air temp $= \theta_0 = 25°$.

$\theta = 140, \ t = 0.$

$\theta = 80, \ t = 20.$

$\theta = 35, \ t = ?$

Step 1 : find c

$$\boxed{\theta - \theta_0 = ce^{-kt}} \quad \text{—①}$$

$$① \Rightarrow 140 - 25 = ce^{-k(0)}$$

$$\Rightarrow 115 = ce^{0}$$

$$115 = c(1).$$

$$\boxed{c = 115.}$$

Step 2: find k.

$$\boxed{\theta - \theta_0 = ce^{-kt}}$$

$$80 - 25 = 115\, e^{-k(20)}.$$

18.

$$80-25 = 115e^{-k(20)}.$$

$$55 = 115e^{-20k}.$$

$$e^{-20k} = \frac{55}{115}.$$

$$e^{-20k} = 0.4783.$$

$\log_e a =$.

Apply log on B.S

$$\log \cancel{e}^{-20k} = \log(0.4783)$$

$$-20k = -0.7395.$$

$$k = \frac{-0.7395}{-20}.$$

$$\boxed{k = 0.037.}$$

Step 3 $\boxed{\text{Find } t}$.

$$\boxed{\theta - \theta_0 = ce^{-kt}}.$$

$$35-25 = 115e^{-0.037 \cdot t}.$$

$$\Rightarrow 10 = 115e^{-0.037t}.$$

$$e^{-0.037t} = \frac{10}{115}.$$

$$e^{-0.037t} = 0.087.$$

log on B.S.

$-0.037t = \log(0.087)$.

$= -0.037t = -2.442$.

$$t = \frac{2.442}{0.037} = 66.$$

$$\boxed{t = 66}$$

Law of Natural Growth or decay.

Let $x(t)$ be the amt of substance at time t.
If the rate of change of substance is prop to the amt of substan available at that time

$x(t)$ sust.

$$\frac{dx}{dt} \propto x.$$

$$\frac{dx}{dt} = kx.$$

Natural growth

If k is +ve.

$$\frac{dx}{dt} = kx.$$

$$\int \frac{dx}{x} = \int k\,dt.$$

$$\log x = kt + \log c$$

$$\log x - \log c = kt$$

$$\log\left(\frac{x}{c}\right) = kt$$

$$\frac{x}{c} = e^{kt}$$

$$x = Ce^{kt}$$

If k is −ve.

$$\frac{dx}{dt} = -kx.$$

$$\boxed{x = ce^{-kt}}$$

Natural decay

A bacterial culture, growing +ve exponentially increase from 100 to 400 gm in 10 hrs. How much was present after 3 hrs from unit a.

at $t=0$, $N=100$.

$t=10$, $N=400$

$t=3$, $N=?$

find C

$$\boxed{N = ce^{kt}} \quad -(1)$$

$$100 = ce^{k(0)}$$

$$\boxed{c = 100}$$

find k

$$400 = 100\, e^{k(10)}$$

$$e^{k(10)} = \frac{400}{100}$$

$$e^{10k} = 4$$

L.O.B.S.

$$\log e^{10k} = \log 4$$

$$10k = \log 4$$

$$\boxed{k = \frac{1}{10}\log 4}$$

$N = ce^{kt}$

$= 100e^{\frac{1}{10}\log 4(3)}$

$= 100e^{\frac{3}{10}\log 4}$

$= 100\ 4^{\frac{3}{100}}$

$N = 125.$

O.D.E.

M2 Differential Equations

1. Change terms.
2. S.O.B.S.
3. Algebraic formula
4. eqn.
5. higest order.
6. highest degree

① $y = x \cdot \dfrac{dy}{dx} + \sqrt{1+\left(\dfrac{dy}{dx}\right)^2}$.

$\left(y - x \cdot \dfrac{dy}{dx}\right) = \sqrt{1+\left(\dfrac{dy}{dx}\right)^2}$

S.O.B.S

$\left(y - x \cdot \dfrac{dy}{dx}\right)^2 = \left(\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}\right)^2 \qquad (a-b)^2$

$y^2 + x^2 \dfrac{d^2y}{dx^2} - 2 \cdot \cancel{(y)}\left(x^2 \dfrac{d^2y}{dx^2}\right)$

$y^2 + x^2 \dfrac{d^2y}{dx^2} - 2xy \dfrac{dy}{dx} = 1 + \left(\dfrac{dy}{dx}\right)^2$

$\Rightarrow \boxed{(y^2-1) + x^2 \dfrac{d^2y}{dx^2} - 2xy \dfrac{dy}{dx} - \left(\dfrac{dy}{dx}\right)^2 = 0}$

higest derivative order 2.

power/degree = 1.

Exact Differential Equations

$M\,dx + N\,dy = 0$ — (1).

$M(x,y)\,dx + N(x,y)\,dy = 0$.

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

(1) is Exact D.E.

$$\int M\,dx + \int (\text{terms of } N \text{ without } x)\,dx = c.$$

(y - const).

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(1). $\underbrace{(e^{y}+1)\cos x}_{M}\,dx + \underbrace{e^{y}\sin x}_{N}\,dy = 0$ — (1)

$\boxed{\text{dont do } uv}$

eqn (1) is of form $M\,dx + N\,dy = 0$.

$M = (\overset{u}{e^{y}+1})\,\overset{v}{\cos x}$.

$N = (e^{y}\sin x)$.

~~cosx = cosx~~

$\cancel{\frac{\partial M}{\partial y} = e^{y}(\sin x)\cos x\,(e^{y})}$

$\frac{\partial N}{\partial x} = e^{y}\sin x$.

$\frac{\partial N}{\partial x} = e^{y}\cos x$.

$\frac{\partial M}{\partial y} = e^{y}\cos x + \cos x$.

$\frac{\partial M}{\partial y} = e^{y}\cos x + 0$.

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

$\int M dx + \int$ (terms of N without x) dy) $dy = c$.
(y const)

$\int (e^y \cos x + \cos x) dx + \int 0 dy = c$. $\quad \int \cos x = \sin x$

$e^y \int \cos x dx + \int \cos x dx + \int 0 dy = c$.

~~$e^y \int \sin x + \int \sin$~~

$e^y \sin x + \sin x = c$

$\sin x (e^y + 1) = c$.

$\boxed{\frac{\partial xy}{\partial y} = x(1)}$

(2). $(y^2 - 2xy) dx = (x^2 - 2xy) dy$ — (1) $\Rightarrow (y^2 - 2xy) dx + (2xy + x^2) dy = 0$ — (1).

$M dx + N dy = 0$.

$M = y^2 - 2xy$

$\frac{\partial M}{\partial y} = 2y - 2x(1) = 0$.

$N = \cancel{x^2 - 2xy}$ $2xy + x^2$

$\frac{\partial N}{\partial x} = \cancel{2x - 2x = 0}$ $2y + 2x$

Both or ~~not~~ same

(4)

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ = Exact.

$\int M\,dx + \int N\,dy = C$.

y const   No x terms

$\int (y^2 - 2xy)\,dx + \int (2xy - x^2)\,dy = C$.

$y^2 \cancel{ef}$.

$\Rightarrow \int (y^2 - 2xy)\,dx + \int 0\,dy = c$.

$\int -x\,dx + \int 0\,dy = c$

$-\frac{2x^2}{2} = c$.

$= c = -x^2$

$\boxed{c - x^2 = 0}$ ans.

$\boxed{\int x = \frac{2x^2}{2}}$

$y^2 \int 1\,dx - 2y \int x\,dx = c$.

$y^2 x - 2y \cdot \frac{x^2}{2} = c$

$\boxed{y^2 x - y x^2 = c}$

Solve. $\left[y\left(1+\frac{1}{x}\right)+\cos y\right]dx+(x+\log x - x\sin y)dy$

$M = y\left(1+\frac{1}{x}\right)+\cos y$

$\frac{\partial M}{\partial y} = y + \frac{y}{x} + \cos y$

$\frac{\partial M}{\partial y} = 1 + \frac{1}{x} - \sin y$

$N = x + \log x - x\sin y$

$\frac{\partial N}{\partial x} = 1 + \frac{1}{x} - 1\sin y$

$\frac{\partial N}{\partial x} = 1 + \frac{1}{x} - \sin y$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ = exact

$\int M dx + \int N dy = c$

y const   no 'x' terms

$\int\left(y + \frac{x}{y} + \cos y\right)dx + \int x + \log x - x\sin y = c$

$\int\left(y + \frac{x}{y} + \cos y\right)dx + \int 0\,dx = c$

$\cancel{y\int}$

$y\int 1\,dx + y\int \frac{1}{x}dx + \int \cos y\,dx = c$

$xy + y\log x + x\cos y = c$

$xy + y\log x + x\cos y = c$

(4)

$$\begin{array}{r} 5 \\ 15 \\ 10 \\ \hline 30 \end{array}$$

Non Exact D.E

Method ①: Inspection Method. (uv)

① $d(xy) = xdy + ydx$.

② $d\left(\frac{x}{y}\right) = \frac{ydx - xdy}{y^2}$.

Aliens Kdork

$\frac{\text{deno (num)} - \text{Num}(d}{\text{deno}^2}$.

③ $d\left(\frac{y}{x}\right) = \frac{xdy - ydx}{x^2}$

④ $d\left(\frac{x^2+y^2}{2}\right) = xdx + ydy$.

⑤ $d\left[\log\left(\frac{y}{x}\right)\right] = \frac{xdx - ydy}{xy}$.

⑥ $d\left[\log\left(\frac{x}{y}\right)\right] = \frac{ydx - xdy}{xy}$.

⑦ $d\left[\tan^{-1}\left(\frac{x}{y}\right)\right] = \frac{ydx - xdy}{x^2+y^2}$

⑧ $d\left[\tan^{-1}\left(\frac{y}{x}\right)\right] = \frac{xdy - ydx}{x^2+y^2}$

⑨ $d\left[\log(xy)\right] = \frac{ydx - xdy}{xy}$.

⑩ $d\left[\log(x^2+y^2)\right] = \frac{2(xdx+ydy)}{x^2+y^2}$

⑪ $d\left[\frac{e^x}{y}\right] = \frac{ye^x dx - e^x dy}{y^2}$

Solve $(1+xy)x\,dy + (1-yx)y\,dx = 0$ — ①.

$M = (1-yx)y\,dx$.

$\frac{\partial M}{\partial y} = y - y^2x\,dx$.

$\frac{\partial M}{\partial y} = x$

$N = (1+xy)x\,dy$.

$\frac{\partial N}{\partial x} = x + x^2y\,dy$.

$\frac{\partial N}{\partial x} = y$.

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

∴ There are Non exact Ee non diff eans .

Take ean ①.

$(1+xy)x\,dy + (1-yx)y\,dx = 0$

$\boxed{\text{Multiply}}$

$\cancel{xdy} + x^2y\,dy + y - y^2x\,dx = 0$

~~Take xy common~~

$\boxed{\text{put square terms one side}}$.

~~xy(~~

$x^2y\,dy - y^2x\,dx + x\,dy + y\,dx = 0$.

$\boxed{\text{Take } xy \text{ common}}$

$xy(x\,dy - y\,dx) + x\,dy + y\,dx = 0$.

(8)

$\boxed{\text{divide by } x^2y^2}$

$$\frac{xy(xdy - ydx)}{x^2y^2} + \frac{xdy + ydx}{x^2y^2} = 0.$$

$$\Rightarrow \frac{xdy - ydx}{x^2y} + \frac{xdy + ydx}{x^2y^2} = 0.$$

$\boxed{\text{Square terms one side}}$

$$\frac{xdy + ydx}{x^2y^2} + \frac{xdy - ydx}{xy} = 0. \longrightarrow \frac{\cancel{x}dy}{\cancel{x}y} - \frac{\cancel{y}dx}{x\cancel{y}}$$

$\boxed{\text{Apply formulae}}$ $\frac{dy}{y} - \frac{dx}{x}$

$\frac{1}{y}dy - \frac{1}{x}dx.$

$$\frac{d(xy)}{(xy)^2} + d\left[\log\left(\frac{y}{x}\right)\right]$$

$\boxed{\text{Apply Integration}}$

$$\int \frac{d(xy)}{(xy)^2} + \int \frac{1}{y}dy - \int \frac{1}{x}dx.$$

$\frac{d(xy)}{(xy)^2}$

$-\frac{1}{xy}.$

$$-\frac{1}{xy} + \log y - \log x = 0.$$

$\frac{1}{y} = \log$

$\frac{1}{x} = \log$

Q) Solve $x\,dx + y\,dy = \frac{x\,dy - y\,dx}{x^2+y^2}$ — (1).

$$d\left(\frac{x^2+y^2}{2}\right) = d\left(\tan^{-1}\left(\frac{y}{x}\right)\right)$$

I.O.B.S.

$$\int d\left(\frac{x^2+y^2}{2}\right) = \int d\left(\tan^{-1}\left(\frac{y}{x}\right)\right)$$

$$\frac{x^2+y^2}{2} = \tan^{-1}\frac{y}{x}$$ //.

Integrating factor

$$M\,dx + N\,dy = 0 \longrightarrow M_1\,dx + N_1\,dy = 0.$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}. \qquad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

IF.

4 METHODS

(1) INSPECTION METHOD.

② $M(x,y)\,dx + N(x,y)\,dy = 0$ is a homo DE

$$IF = \frac{1}{Mx+Ny}$$

Prob

① Solve $(x^2y\,dx - (x^3+y^3)\,dy = 0$.

$M = x^2y$. | $N = -(x^3+y^3)$.

$\frac{\partial M}{\partial y} = x^2$. | $\frac{\partial N}{\partial x} = -3y^{2}$.

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$\therefore$ Eqn ① is non exact.

eqn ① is a homogenius Diff eq^n.

Then the $IF = \frac{1}{Mx+Ny}$.

$$= \frac{1}{x^2y - x^3 - y^3}$$

$Mx + Ny$

$\therefore \dfrac{1}{x(x^2y) - (x^3 - y^3)y}$

$\dfrac{1}{x^3y - y^3x + y^3}$

$\therefore IF = \dfrac{1}{y^3}$

① × I.F.

$\dfrac{1}{y^3} \times x^2y\,dx - (x^3 + y^3) \times \dfrac{1}{y^3}\,dy = 0$

$\Rightarrow \dfrac{x^2}{-y^2}\,dx + \left(\dfrac{-x^3}{y^3} + 1\right)dy = 0$ — ②

$\dfrac{\partial M}{\partial y} = -x^2(-2)y^{-3} \quad 2x^2y^3$

$= \dfrac{2x^2}{y^4} \quad \Big| \quad \dfrac{\partial N}{\partial x} = -\dfrac{3x^2}{y^3} + 0$

$\dfrac{\partial M_1}{\partial y} = \dfrac{\partial M_1}{\partial x}$

$\int M dx + \int N dy = c$
y const

=

$$\int \frac{-x^2}{y^3} dx + \int \frac{1}{y^3} dy = c.$$

$$\Rightarrow \frac{-1}{y^3} \int x^2 dx + \int y^{-3} dy = c.$$

$$\int x^n = \frac{x^{n+1}}{n+1} \quad = \frac{-1}{y^3}\left(\frac{x^{2+1}}{2+1}\right) + \frac{y^{-3+1}}{-3+1} = c.$$

$$= \frac{-x^3}{2y^3} - \frac{y^{-2}}{-2} = c.$$

$$\frac{-x^3}{2y^3} + \frac{2}{y^2} = c.$$

Powers are equal is called homogenius.

eg. $(x^2y^1)dx + (x^1y^2)dy$
3 3.

$$(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0.$$

$$M\,dx + N\,dy = 0.$$

$$(x^2y - 2xy^2)\,dx + (-x^3 + 3x^2y)\,dy = 0 \quad \text{— (1)}$$

$$M = x^2y - 2xy^2 \qquad\qquad N = -x^3 + 3x^2y$$

$$\frac{\partial M}{\partial y} = x^2 - 2x(2y) \qquad\qquad \frac{\partial N}{\partial x} = -3x^2 + 6xy.$$

$$= x^2 - 4xy.$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \quad \text{not exact.}$$

But eqn (1) is homogenus

$$IF = \frac{1}{Mx + Ny} = \frac{1}{x(x^2y - 2xy^2) + y(-x^3 + 3x^2y)}.$$

$$= \frac{1}{\cancel{x^3y} - 2x^2y^2 - \cancel{x^3y} + 3x^2y^2}$$

$$IF = \frac{1}{x^2y^2}.$$

14

$\boxed{\text{Multiply IF} \times \text{eqn (1)}}$

$IF = \dfrac{1}{x^2y^2}$

$$\therefore \left(\frac{x^2y - 2xy^2}{x^2y^2}\right)dx + \left(\frac{-x^3 + 3x^2y}{x^2y^2}\right)dy = 0$$

$$\left[\frac{1}{y} - \frac{2}{x}\right]dx + \left[\frac{-x}{y^2} + \frac{3}{y}\right]dy = 0 \quad \text{(2)}$$

eqn (2) is of the form $M_1 dx + N_1 dy = 0$.

$M_1 = \left[\frac{1}{y} - \underbrace{\frac{2}{x}}_{\text{const}}\right]$ | $N_1 = \left[-\underbrace{\frac{x}{y^2}}_{\text{const}} + \underbrace{\frac{3}{y}}_{\text{const}}\right]$

$\dfrac{\partial M}{\partial y} = \cancel{\log_e y}\ \dfrac{-1}{y^2}$ | $\dfrac{\partial N}{\partial x} = -\cancel{\dfrac{9}{y^9}}\dfrac{1}{y^2}$

$$\therefore \boxed{\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}}$$

$\boxed{\text{Seperate write dont write const term}}$

$\int\limits_{y\,\text{const}} M_1 dx + \int \cancel{\partial f d}(\text{terms of } N_1 \text{ without } x)\, dy = 0$.

$$\int\left(\frac{1}{y} - \frac{2}{x}\right)dx + \int\frac{3}{y}dy = c$$

$\frac{x}{y} - 2\log x + 3\log y = c$

$= \frac{x}{y} - \log x^2 + \log y^3 = c.$ $\quad (\log a - \log b = \log\frac{a}{b})$

$\boxed{= \frac{x}{y} + \log\frac{y^3}{x^2} = c.}$ //.

$r(\theta^2+r^2)d\theta - \theta(\theta^2+2r^2)dr = 0.$

$\Rightarrow (r\theta^2+r^3)d\theta + (-\theta^3-2r^2\theta)dr = 0$

put $\theta = x$, $r = y$.

$(yx^2+y^3)dx + (-x^3-2y^2x)dy = 0$ — ①.

ean ① is of the form $Mdx+Ndy = 0$.

$M = (yx^2+y^3)$ | $N = (-x^3-2y^2x)$.

$\frac{\partial M}{\partial y} = x^2+3y^2.$ | $\frac{\partial N}{\partial x} = -3x^2-2x.$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}.$

∴ It is non exact D.E.

(16)

∴ eqn (1) is non exact & it is a homogeneous DE.

$$IF = \frac{1}{Mx + Ny}$$

$$= \frac{1}{x(yx^2+y^3) + y(-x^3-2y^2x)}$$

$$= \frac{1}{\cancel{x^3y} + 1xy^3 - \cancel{x^3y} - 2y^3x} = \frac{1}{-1y^3x}$$

$$I.F. = \frac{-1}{y^3x}$$

$$M_1dx + N_1dy = 0 \quad -(2)$$

$\boxed{IF \times \text{ eqn (1)}}$

$$\frac{1}{-y^3x} \times (yx^2+y^3)dx + (-x^3-2y^2x)dy = 0$$

$$\therefore \left[\frac{yx^2+y^3}{-y^3x}\right]dx + \left[\frac{-x^3-2y^2x}{-y^3x}\right]dy = 0.$$

$$\left[-\frac{x^2}{y^2} - \frac{1}{y}\right]dx + \left[\frac{x^2}{y^3} + \frac{2}{y}\right] = 0 \quad (2)$$

$M_1 dx + N_1 dy = 0$

$M_1 = -\frac{x^2}{y^2} - \frac{1}{y}$

$N = \frac{x^2}{y^3} + \frac{2}{y}$

$\frac{1}{y^2} = y^{-2}$

$\frac{\partial M}{\partial x} = \frac{2x^3}{y^3} - 0$

$\frac{\partial N}{\partial y} = \frac{2x^3}{y^3} + 0$

$-2y^{-2-1}$

$= -\frac{2}{y^3}$

$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$

$\therefore$ eqn ② is exact.

$\int M_1 dx + \int$ (terms of N_1 without x). $dx = c$

y const

$\int\left(-\frac{x^2}{y^2} - \frac{1}{y}\right)dx + \int\left(\cancel{\frac{x^2}{y^3}} + \frac{2}{y}\right) = c$

$\Rightarrow \int\left(-\frac{x^2}{y^2} - \frac{1}{y}\right) dx + \int \frac{2}{y} dy = c.$

(y const

$y^2 \; \frac{-x^4}{4} - \log x + 2\log y = 0$

$\frac{\theta^4}{4r^2} + \log\left(\frac{r^2}{\theta}\right) = c$

$\frac{x^4}{4y^2} + \log y - \log x = c.$

$\Rightarrow \frac{x^4}{4y^2} + \log\left(\frac{y^2}{x}\right) = c.$

M2 Unit 2.

Higher Order Ordinary Diff Eqns:

The general form of linear Diff eqn. with constant co-eff of order 'n'

is $\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots P_n y = Q(x)$

$$\boxed{\frac{d}{dx} = D.}$$

$$D^n y + P_1 D^{n-1} y + P_2 D^{n-2} y + \dots P_n(y) = Q(x)$$

$$\Rightarrow (D^n + P_1 D^{n-1} + P_2 D^{n-2} + \dots P_n) y = Q(x).$$

$$= f(D) y = Q(x).$$

$$y = y_c + y_p.$$

$$f(D) y = 0.$$

$$\parallel$$

$$m.$$

Auxilliary $f(m) = 0.$

case ① Real & Diff (m_1, m_2).

$$y_c = c_1 e^{m_1 x} + c_2 m_2 e^{m_2 x}.$$

case ② Real & Equal $(m = m_1 = m_2)$.

$$y_c = (c_1 + c_2 x) e^{mx}.$$

case ③ Imaginery & Diff. $(\alpha \pm i\beta)$

$$y_c = e^{\alpha x} (c_1 \cos\beta x + c_2 \sin\beta x).$$

case ④ Imaginery & Equal $(\alpha \pm i\beta)(\alpha \pm i\beta)$.

$$y_c = e^{\alpha x} \left[(c_1 + c_2 x) \cos\beta x + (c_3 + c_4 x) \sin\beta x \right].$$

① Solve $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$.

$D^2y - 8Dy + 15y = 0$.

$(D^2 - 8D + 15)y = 0$ — ①.

eqn ① is of the form $f(D)y = 0$.

∴ Auxilliary eqn is $f(m) = 0$.

$f(D) = D^2 - 8D + 15$.

$= f(m) = m^2 - 8m + 15 = 0$.

roots $\boxed{m = 3, 5.}$

∴ The roots 3 & 5 are real & different.

∴ $GS = y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x}$.

$y_c = c_1 e^{3x} + c_2 e^{5x}$.

② Solve $\frac{d^2y}{dx^2} - a^2y = 0$.

$D^2y - a^2y = 0$.

$(D^2 - a^2)y = 0$. — ①.

eqn ① is of form $f(D)y = 0$

$\Rightarrow$ Auxilliary eqn is $f(m)=0$.

$= m^2 - a^2 = 0$.

$m^2 = a^2$.

$m = \sqrt{a^2}$.

$m = \pm a$.

$m = +a, -a$.

$\therefore$ as The roots are real & ~~equal~~ not equal.

~~CF=~~ ~~$(c_1 + c_2 x)e^{mx}$~~

$$\boxed{y_c = c_1 e^{-ax} + c_2 e^{ax}.}$$ //.

③. $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$.

$D^2y + Dy + y = 0$.

$(D^2 + D + 1)y = 0$ — ①.

$f(m) = 0$.

$$\boxed{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.}$$

$\Rightarrow AE = f(m) = 0$.

$m^2 + m + 1 = 0$.

$(m+1)^2 = 0$.

$= . \ 1 \pm \sqrt{1-4}, \ \frac{-1}{2} \pm i\frac{\sqrt{3}}{2}$

~~m^2~~

~~m..~~

~~$(m+1)^2$~~

~~$m^2 + 1m + 1$.~~

~~$m(m+1) + 1(m+1)$~~

~~$m^2 2m + 1$~~

which is in the form of $\alpha \pm i\beta$.

$$y_c = e^{\alpha x}\left[c_1 \cos \beta x + c_2 \sin \beta x\right].$$

$$\boxed{y_c = e^{-\frac{x}{2}}\left[c_1 \cos \frac{\sqrt{3}}{2} x + c_2 \sin \frac{\sqrt{3}}{2} x\right].}$$

(4). Solve $(D+2)(D-1)^2 y = e^{-2x} + \sinh x$. 2 term find PI.

$$(D+2)(D-1)^2 y = 0$$

$$f(D)\, y = 0.$$

$\Rightarrow$ AE $= f(m) = 0$.

$$(m+2)(m-1)^2 = 0.$$

$$m = (-2), 1, 1$$

$\Rightarrow \therefore$ 2 roots are real & equal

$\Rightarrow$ $y_c = (c_1 + c_2 x)\, e^{mx}$.

$$\boxed{y_c = (c_1 + c_2 x)\, e^{1x}}$$

$$\therefore \boxed{y_c = (c_1 + c_2 x)\, e^{x} + c_3 e^{-2x}}$$

$\Rightarrow$ $y_p =$. $f(D)\, y = Q(x)$.

$$y_p = \frac{1}{f(D)} Q(x)$$

becomes 0.
to derivate

e^{-1x}

$-1)^2$

e^{-x}

$x(-1)$

$1-1)^2$

$y_P = f(D)y = Q(x)$.

$y_P = \frac{1}{f(D)} = Q(x)$.

$y_P = \frac{1}{f(D)} \cdot e^{ax}$, put $D=a$.

$(D+2)(D-\cancel{2})^2 y = e^{-2x} + \cancel{2}\left[\frac{e^{x}-e^{-x}}{\cancel{2}}\right]$.

$(D+2)(D-1)^2 y = e^{-2x} + e^{x} - e^{-x}$.

$\therefore y_P = \frac{1}{f(D)} Q(x)$

$= \frac{1}{(D+2)(D-1)} (e^{-2x} + e^{x} - e^{-x})$.

$= \frac{1}{(D+2)(D-1)^2} \cdot e^{-2x} + \frac{1}{(D+2)(D-1)} e^{1x} - \frac{1}{(D+2)(D-1)} \cdot e^{-1x}$.

put $D=a$.

$\cancel{\frac{1}{(-2+2)(-2-1)^2}} e^{-2x} + \cancel{\frac{1}{(1+2)(1-1)}} e^{x} - \cancel{\frac{1}{(-1+2)(-1-1)}}$

put -2 in $D+2$ it becomes 0.
so vanish so do derivate

$$\frac{1}{(D+2)(D-1)^2}e^{-2x} + \frac{1}{(D+2)(D-1)^2}e^{1x} - \frac{1}{(D+2)(D-1)^2}e^{-1x}$$

1st derivative

$$\frac{x}{1(2(D-1))}e^{-2x} + \frac{x}{1(2(D-1))}e^{x} - \frac{1}{(D+2)(D-1)^2}e^{-x}$$

(-2)

$$- \frac{1}{(-1+2)(-1-1)^2}e^{-1}$$

so go for 2nd derivative

$$-\frac{x}{6}e^{-2x} + \frac{x}{0}$$

$2(0)$

$$-\frac{x}{6}e^{-2x} + \frac{x^2}{2(1)}e^{x} - \frac{1}{4}e^{-x}$$

$$\boxed{y_p = -\frac{x}{6}e^{-2x} + \frac{x^2}{2}e^{x} - \frac{1}{4}e^{-x}}$$

G.S. $\boxed{\therefore y = y_c + y_p}$

$$\therefore y = (c_1 + c_2x)e^{x} + c_3e^{-2x} - \frac{xe^{-2x}}{6} + \frac{x^2e^{x}}{2} - \frac{e^{-x}}{4}$$

x.

$(D^3 - 6D^2 + 11D - 6)y = e^{-2x} + e^{-3x}$.

$AE = (D^3 - 6D^2 + 11D - 6)y = 0$.

$1, 3, 2$.

The roots are real & not equal.

$c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$.

$$\boxed{y_c = c_1 e^{x} + c_2 e^{3x} + c_3 e^{2x}}$$

$y_p = \frac{1}{f(D)} = Q(x)$.

$y_p = \frac{1}{f(D)} = e^{ax}$.

put $D = a$.

$\frac{1}{D^3 - 6D^2 + 11D - 6} (e^{-2x} + e^{-3x})$

$y = y_c + y_p$.

$c_1 e^{x} + c_2 e^{3x} + c_3 e^{2x}$

$- \frac{e^{-2x}}{\cancel{4}60} - \frac{e^{-3x}}{\cancel{4\,16}120}$.

$\frac{1}{D^3 - 6D^2 + 11D - 6} \cdot e^{-2x} + \frac{1}{D^3 - 6D^2 + 11D - 6} \cdot e^{-3x}$.

put $D = -2$ &
$D = -3$

$\frac{1}{-160} \cdot e^{-2x} + \frac{1}{-16} \cdot e^{-3x}$.

$$\boxed{y_p = \frac{e^{-2x}}{-\cancel{4}60} + \frac{e^{-3x}}{\cancel{4\,16}\,120}}$$

Solve $(4D^2 - 4D + 1)y = 100$. $\qquad 100 = e^{0x}$.

$\therefore (4D^2 - 4D + 1)y = 100e^{0x}$.

$AE = 4D^2 - 4D + 1 = 0$. $\qquad f(m) = 0$.

$\frac{1}{2}, \frac{1}{2}$

$=$

The roots are real & equal $\qquad c_1 e^{m_1 x}$.

$\left[(c_1 + c_2 x)e^{mx} + \cancel{c_3 x} \; c_3 e^{m_1 x}\right]$

$\boxed{y_c = (c_1 + c_2 x)e^{\frac{1}{2}x}}$

$y_p = \frac{1}{f(D)} \times Q(x)$.

$\frac{1}{f(D)} \times e^{ax}$.

$\frac{1}{(4D^2 - 4D + 1)} \times 100e^{0x}$. $\qquad \frac{1}{4(0)^2 - 4(0) + 1}$. $\quad \frac{1}{1}$

$\boxed{\text{put } D = a}$ where $\boxed{a = 0}$

$\frac{1}{4(0)^2 - 4(0) + 1} \cdot 100e^{0x}$.

$\Rightarrow 100\,e^{0x}$

$= 100(1) \Rightarrow 100 \Rightarrow \boxed{\therefore y_p = 100}$

$\boxed{GS = y_c + y_p}$

Particular Integral.

$y = y_c + y_p$.

① $\frac{1}{D} x = \int x = \frac{x^2}{2}$.

② $\frac{1}{D^3} \cos x = \int \cdots$

Working Rule:

① $PI = \frac{1}{f(D)} \cdot Q(x)$

$= \frac{1}{f(D)} \cdot e^{ax}$.

$= \frac{1}{f(a)} \cdot e^{ax}$

$= \frac{1}{f(a)} e^{ax}, \; f(a) \neq 0$

② Let $f(a) = 0$.

Then $PI = \frac{x}{f'(D)} e^{ax}$.

$\vdots$

$PI = \frac{x^2}{f''(D)} \cdot e^{ax}$.

① Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{2x}$.

$\Rightarrow$ AE $\Rightarrow$ $D^2y + 4Dy + 3y = 0$.

$(D^2+4D+3)y = 0$.

$\Rightarrow f(m)=0$ $m^2+4m+3 = 0$.

$\Rightarrow$ $m = -1, -3$.

$\Rightarrow$ roots are real & not equal.

$c_1 e^{m_1 x} + c_2 e^{m_2 x}$.

$\boxed{y_c = c_1 e^{-x} + c_2 e^{-3x}}$

$y_P = \frac{1}{f(D)} \cdot Q(x)$

$= \frac{1}{f(D)} \cdot e^{ax}$.

$\frac{1}{D^2+4D+3} \cdot e^{2x}$.

$2^2+4(2)+3$.

$4+8+3$

$\boxed{\text{put } D = a = 2}$

$\boxed{y_P = \frac{e^{2x}}{15}}$

$\boxed{GS = c_1 e^{-x} + c_2 e^{-3x} + \frac{e^{2x}}{15}}$

Solve $y'' - y' - 2y = 3e^{2x}$, $y(0) = 0,\ y'(0) = 2$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 3e^{2x}.$$

$\Rightarrow$ AE $= D^2y - Dy - 2y = 0.$

$= (D^2 - D - 2)y = 0.$

$\Rightarrow \quad m^2 - m - 2 = 0$

$\Rightarrow \quad \boxed{m = -2, -1}$

$\Rightarrow$ The roots are real & not equal.

$\Rightarrow\ C_1e^{m_1x} + C_2e^{m_2x}.$

$$\boxed{y_c = C_1e^{-2x} + C_2e^{-x}.}$$

$$y_P = \frac{1}{f(D)} \cdot Q(x)$$

$$= \frac{1}{f(D)} \cdot e^{ax}$$

$4 - 2 - 2.$ 0.

$$\frac{1}{D^2 - D - 2} \cdot 3e^{2x}.$$

$\frac{d}{dx}(x^2 - x - 2).$

$D = a = 2.$

$2x - 1$

$\boxed{D'} \Rightarrow \frac{x}{f'(D)}e^{ax}$

$$\frac{x}{2D - 1} \cdot 3e^{2x} \Rightarrow \frac{x}{3} \cdot 3e^{2x} \Rightarrow \boxed{xe^{2x}}$$

$$GS = C_1e^{-2x} + C_2e^{-x} + xe^{2x}$$

If $y(0) = 0$.

$y(x_0) = y_0$.

$$\boxed{y_0 = c_1 e^{-x_0} + c_2 e^{-2x_0} + x_0 e^{2x}}$$

$$0 = c_1 e^{-0} + c_2 e^{-2(0)} + 0e^{0}$$

$$\therefore \boxed{c_1 + c_2 = 0.} \quad -(1)$$

If $y'(0) = -2$.

$$y' = -c_1 e^{-x} + 2c_2^{2x} + 2x e^{2x} + e^{2x}.$$

$$y_0' = -c_1 e^{-x_0} + 2c_2 e^{2x_0} + 2x_0 e^{2x_0} + e^{2x_0}$$

$$-2 = -c_1 + 2c_2 + 0 + 1$$

$$\therefore \Rightarrow \boxed{-c_1 + 2c_2 = -3.} \quad (2)$$

$-c_1 + 2c_2 = -3.$

$c_1 + c_2 = 0.$

Jov

$$\boxed{\begin{aligned} c_1 &= -1. \\ c_2 &= 1. \end{aligned}}$$

$$\therefore y = -e^{-x} + e^{2x} + xe^{2x}.$$

The diff eqn $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = -2\cosh x$.

$y=0,\ y'(0)=1$

$D^2y + 4D + 5 = -2\frac{(e^x + e^{-x})}{2}$.

$D^2y + 4Dy + 5y = e^x - e^{-x}$.

$\Rightarrow$ AE $= D^2y + 4D + 5 = 0$.

$m^2y + 4m + 5 = 0$.

$(m^2 + 4m + 5)y = 0$.

$\Rightarrow m = -2+i,\ -2-i \Rightarrow \boxed{-2 \pm i}$ $\qquad \alpha \pm i\beta$.

The roots are imaginary & not equal.

$C_1 E C_2 x$

a_1

$$\boxed{y_c = e^{-2}[c_1 \cos\beta x + c_2 \sin\beta x].}$$

$\frac{1}{f(D)y} \cancel{x}. Q(x)$

$\frac{1}{f(D)} \cdot e^{ax}$.

$\frac{1}{D^2+4D+5} \cdot -(e^x + e^{-x})$.

$\frac{1}{D^2+4D+5} \cdot e^{1x} + e^{-1x}$.

$\frac{e^x}{D^2+4D+5} + \frac{e^{-x}}{D^2+4D+5}$.

$D = a = 1,\quad D = a = -$

$\frac{e^x}{10} + x.\frac{e^{-x}}{f'(D)y}$.

$\frac{x}{2x+4} \cdot e^{-x}$.

x^2+4x+5.

$2x+4$.

$x=-1$.

$-2+4=2$.

$\frac{x}{2} \cdot e^{-x}$

$\therefore \frac{e^{x}}{10} \mp \frac{xe^{-x}}{2}$.

$\therefore y = y_c + y_p$

$\boxed{GS = e^{-2x}\left[c_1\cos\beta x + c_2 \sin\beta x\right] \mp \frac{e^{x}}{10} \mp \frac{xe^{-x}}{2}}$.

If $y(0)=0$.

$y(x_0) = y_0$ in eq (2)

$\boxed{y'(0)=1}$ diff

$y' = e^{-2x}\left[-c_1\sin x + c_2\cos x\right) - 2e^{-2x}\left(c_1\cos x + c_2 \sin x\right) - \frac{e^{x}}{10} + \frac{e^{-x}}{2}$. (2).

$0 = 1\left[c_1 + 0\right] - \frac{1}{10} - \frac{1}{2}$.

$\boxed{c_1 = \frac{3}{5}}$ $y'(0)=1$, $y'(x_0) = y_0$ in eq (3).

$\boxed{c_2 = \frac{9}{5}}$

sub

Type ② Trig

$f(D)y = \sin bx$ or $\cos bx$.

$y_P = \frac{1}{f(D)} \sin bx$.

put only $D^2 = (-b)^2$.

Eg. $\frac{1}{D+2} \sin bx$.

$\boxed{\text{Rationalise}}$

- Solve $(D^2 + 3D + 2)y = \sin 3x$.

$\Rightarrow AE = (D^2 + 3D + 2)y = 0$

$\Rightarrow \quad m^2 + 3m + 2 = 0$.

$\Rightarrow \quad m = -1, -2$

$\Rightarrow$ The roots are real & not equal.

$y_c = \boxed{c_1 e^{-x} + c_2 e^{-2x}}$

$y_p = \frac{1}{f(D)} \cdot \sin bx$

$\frac{1}{D^2 + 3D + 2} \cdot \sin 3x$.

$\boxed{\text{put } D^2 = (-b)^2}$

$\frac{1}{(-3)^2 + 3D + 2} \cdot \sin 3x$.

$\frac{1}{-9 + 3D + 2} \cdot \sin 3x$.

$\frac{1}{3D - 7} \cdot \sin 3x$

$$\frac{1}{3D-7} \sin 3x.$$

$$\frac{3D+7}{(3D-7)(3D+7)} \cdot \sin 3x.$$

$$= \frac{3D+D}{(3D)^2-(7)^2} \sin 3x.$$

$$\frac{3D+7}{9D^2-49} \sin 3x \qquad \boxed{D^2=-9.}$$

$$\frac{3D+7}{9(-9)-49} \sin 3x.$$

$$\frac{3D+7}{130} \sin 3x.$$

$$= \frac{3D \sin 3x + 7 \sin 3x}{130}.$$

$$y_P \quad \frac{(-9\cos 3x + 7\sin 3x)}{130}.$$

$GS = y_c + y_P$

$$\therefore y = c_1 e^{-x} + c_2 e^{-2x} - \frac{(9\cos 3x + 7\sin 3x)}{130}.$$

convert.
$\cos^2 x \to \cos 2x.$

$\cos^2 x = \frac{1+\cos 2x}{2}.$

Solve $(D^2-4)y = 2\cos^2 x.$

$f(D) = D^2-4 = 0.$

$m = 2, -2.$

$$\boxed{y_c = c_1 e^{2x} + c_2 e^{-2x}.}$$

$y_p = \frac{1}{f(D)y} \cdot Q(x)$

$= \frac{1}{f(D)y} \cdot 2\cos^2 x,$

$\frac{1}{D^2-4}\left[\cancel{2}\left[\frac{1+\cos 2x}{\cancel{2}}\right]\right]_{e^{0x}}.$

$\frac{1}{D^2-4} \quad 1 + \cos 2x.$

$\frac{1}{D^2-4} 1 + \frac{1}{D^2-4} \cos 2x.$

$\frac{1}{D^2-4} e^{0x} + \frac{1}{D^2-4} \cos 2x.$

$\boxed{\text{put } D^2=0}$ $\boxed{\text{put } D^2 = 2}$ $\boxed{\text{put } D^2 = -(2)^2}$

$\frac{1}{-4} e^{0x} + \frac{1}{0} \cos 2x.$

$$\frac{1}{-4} e^{0x} + \frac{x}{f'(D)y} \cos 2x$$

$$x^2 - 4$$
$$2x$$

$$\frac{1}{-4} e^{0x} + \frac{x}{2x} \cos 2x$$

$$\frac{1}{-4} e^{0x} + \frac{1}{2} \cos 2x$$

$$\frac{1}{(0)^2-4} e^{0x} + \frac{1}{-4-4} \cos 2x$$

$$\boxed{y_P = -\frac{1}{4} - \frac{1}{8} \cos 2x}$$

Q Solve $(D^2+4)y = e^x + \sin 2x + \cos 2x$.

$f(D) = D^2+4$

$m^2+4=0$

$m = \pm 2i$

$m = 0 \pm 2i$

$m = \alpha \pm i\beta$

$y_c = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$

$= e^{0x}(c_1 \cos 2x + c_2 \sin 2x)$

$$\boxed{y_c = c_1 \cos 2x + c_2 \sin 2x}$$

$y_p = \frac{1}{f(D)} \cancel{x}\, Q(x)$

$\frac{1}{D^2+4} \left(e^x + \sin 2x + \cos \cancel{x}2x\right).$

$\frac{e^{1x}}{D^2+4} + \frac{\sin 2x}{D^2+4}{}^{a=2} + \frac{\cos 2x}{D^2+4}{}^{a=2}$

$\boxed{D^2=1.}$ $\boxed{D^2=-(2)^2}$. $\boxed{D^2=-(2)^2.}$

$\frac{1}{1^2+4} + \frac{1}{0} + \frac{1}{0}.$

$\frac{1}{5}e^x + \frac{x}{f'(D)y} + \frac{x}{f'(D)y}.$

$\frac{\cancel{1}}{5}e^x \cancel{+} \frac{x}{2D} \cdot \sin 2x + \frac{x}{2D} \sin 2x.$

$\frac{e^x}{5} \cancel{+} \frac{x}{2(2)} \cos 2x + \frac{x}{2(2)} \sin 2x.$

$\boxed{\frac{e^x}{5} - \frac{x}{4}\cos 2x + \frac{x}{4}\sin 2x.}$

$y_c + y_p$

$c_1 \cos 2x + c_2 \sin 2x,$

$(D^2-4D+3)y = \sin 3x \cos 2x$.

$2\sin A\cos B =$
$\sin(A+B)+\sin(A-B)$.

$\frac{1}{f(D)}$

$2\sin A\sin B =$.
$\cos(A-B) - \cos(A+B)$

$AE = m^2-4m+3=0$.

$m = 1, 3$.

real & diff

$$\boxed{y = c_1 e^{x} + c_2 e^{3x}}.$$

$y_p = \frac{1}{f(D)} \cdot Q(x)$.

$\frac{1}{D^2+4D+3}$ $\underline{\sin 3x}\ \underline{\cos x}$. ($\sin A\cos B$) form found

Multiply & divide by 2.

$\frac{1}{2(D^2+4D+3)}$ $2(\sin 3x \cos 2x)$

$\frac{1}{2(D^2-4D+3)}$ $\sin(3x+2x)+\sin(3x-2x)$.

Take $\frac{1}{2}$ common

$\frac{1}{2}\left[\frac{\sin 5x}{D^2-4D+3} + \frac{\sin x}{D^2-4D+3}\right]$

$$\frac{1}{2}\left[\frac{\sin 5x}{D^2-4D+3}+\frac{\sin x}{D^2-4D+3}\right].$$

put $D^2=-(5)^2$, $D^2=-(1)^2$.

$=-25$. $=-1$.

$$\frac{1}{2}\left[\frac{\sin 5x}{-25-4D+3}+\frac{\sin x}{-1-4D+3}\right].$$

$$\frac{1}{2}\left[\frac{\sin 5x}{-22-4D}+\frac{\sin x}{2-4D}\right].$$

$$\frac{1}{4}\left[\frac{\sin 5x}{-11-4D}+\frac{\sin x}{1-4D}\right]$$

$-(4D+11)$.

$$\frac{1}{4}\left[\frac{\sin 5x}{\cancel{+}4D+11}+\frac{\sin x}{\cancel{4D+1}\ 1-2D}\right].$$

$$=\frac{1}{4}\left[\frac{4D-11}{(4D+11)(4D-11)}\sin 5x+\frac{\cancel{Si}\,1+2D\sin x}{(1-2D)(1+2D)}\right].$$

$$\frac{1}{4}\left[\frac{4D-11}{16D^2-121}\sin 5x+\frac{(1+2D)(\sin x)}{1-4D^2}\right].$$

[illegible]

$$\frac{1}{4}\left[\frac{-(2D-1)}{\cancel{D}\,4(-25)-121}\sin 5x+\frac{(1+2D)\sin x}{1+4}\right.$$

$\frac{1}{4}\left[\frac{1}{221}(2D-1)\sin 5x + \frac{1}{5}(1+2D)\sin x\right].$

$\frac{1}{884}(2D\sin 5x - \sin 5x) + \frac{1}{20}(\sin x + 2D\sin$.

$\frac{1}{884}\left[\cancel{2}\, 10\cos \underline{5x} - \sin 5x\right] + \frac{1}{20}(\sin x + 2\cos x)$

$GS = y = y_c + y_p$

Type ③.

If $f(D)y = x^k$

Then $y_p = \frac{1}{f(D)} x^k$.

$= \frac{1}{[1+Q(D)]} = [1+Q(D)]^{-1}$

① $\frac{1}{1-D} = (1-D)^{-1} = 1 + D + D^2 + D^3 + \dots$

② $\frac{1}{1+D} = (1+D)^{-1} = 1 - D + D^2 - D^3 + D^4 \dots$

③ $\frac{1}{(1-D)^2} = (1-D)^{-2} = 1 + 2D + 3D^2 + 4D^3 + \dots$

④ $\frac{1}{(1+D)^2} = 1 - 2D + 3D^2 - 4D^3$.

⑤. $\frac{1}{(1-D)^3} = (1-D)^{-3} = 1 + 3D + 6D^2 + 10D^3 + \ldots$

⑥ $\frac{1}{(1+D)} = (1+D)^{-3} = 1 - 3D + 6D^2 - 10D^3 - \ldots$

$(D^2+D+1)y = x^3$. poly.

$f(D)y = x^k$.

$m = \frac{-1}{2} \pm i\frac{\sqrt{3}}{2}$

$\boxed{\alpha \pm i\beta}$

$y_c = e^{\alpha x}[c_1 \cos \beta x + c_2 \sin \beta x]$

$= e^{-1/2 x}\left[c_1 \cos \frac{\sqrt{3}}{2} + c_2 \sin \frac{\sqrt{3}}{2}\right]$.

$f(D)y = x^2$.

$y_p = \frac{1}{D^2+D+1} x^2$.

$= \frac{1}{1+(\quad)} x^2$.

$\frac{1}{1+D^2+D}$

$\left[1+(D^2+D)\right]^{-1} x^2$

$\therefore (1+D)^{-1} = 1-D+D^2-D^3+\cdots$

$\left[1-(D^2+D)+(D^2+D)^2-(D^2+D)^3+\cdots\right.$

$\left[1-D^2-D+D^4+D^2+2D^3\right.$

Unit 4.

① grad → ①
② D.D.
③ Unit normal vector.
④ Angle b/w

② div → solenoidal

③ curl → Irrotational

$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}.$$

$$\text{grad}\,\phi = \nabla\phi = i\frac{\partial\psi}{\partial x} + j\frac{\partial\psi}{\partial y} + k\frac{\partial\psi}{\partial z}.$$

$$f = x^2y + y^2x + z^2.$$

$$\text{grad}\,f = \nabla f = i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}.$$

$$i\frac{\partial}{\partial x}(x^2y + y^2x + z^2) + j\frac{\partial}{\partial y}(x^2y + y^2x + z^2)$$

$$+ k\frac{\partial}{\partial z}(x^2y + y^2x + zy.$$

$$\text{grad}\,f = i(2xy + 2yx) + j(x^2 + 2yx) + k(2z).$$

(1) $r^n = (\_\_\_\_)$

(2) D.D.

(3) unit

∇r^n = grad

$\nabla \times r^n$ = curl

$\nabla \cdot r^n$ = div

*

(1) prove that $\nabla r^n = n r^{n-2} \bar{r}$.

$\bar{r} = xi + yj + zk$ & $r = |\bar{r}|$.

$$\boxed{r^2 = x^2 + y^2 + z^2 \quad \text{—} (1)}$$

diff (1) w.r.t 'x'

$$\cancel{2} r \frac{dr}{dx} = \cancel{2} x$$

$$\boxed{\frac{\partial r}{\partial x} = \frac{x}{r}}$$

diff (1) w.r.t y

$$2r \frac{dr}{dy} = 2y$$

$$\boxed{\frac{\partial r}{dy} = \frac{y}{r}}$$

diff (1) w.r.t z

$$2r \frac{dr}{dz} = 2z$$

$$\boxed{\frac{\partial r}{\partial z} = \frac{z}{r}}$$

let $\nabla r^n = \left(i \frac{\partial r^n}{\partial x} + j \frac{\partial r^n}{\partial y} + k \frac{\partial r^n}{\partial z} \right)$

$$= i \cdot n r^{n-1} \frac{\partial r}{\partial x} + j \cdot n r^{n-1} \frac{\partial r}{\partial y} + k \cdot n r^{n-1} \frac{\partial r}{\partial z}$$

$$= i \cdot n r^{n-1} \left(\frac{x}{r} \right) + j \cdot n r^{n-1} \left(\frac{y}{r} \right) + k \cdot n r^{n-1} \left(\frac{z}{r} \right)$$

$\frac{n r^{n-1}}{r}(xi + yj + zk)$.

$n r^{n-2} \bar{r}$.

LHS = LHS.

Hence proved.

② Show that $\nabla[f(r)] = \frac{f'(r)}{r}\bar{r}$, where.

$\bar{r} = xi + yj + zk$.

(OR)

If $\bar{r}$ is a position vector of point $P(x, y, z)$.

Then prove that $\nabla f(r) = f'(r)\frac{\bar{r}}{r}$.

Sol $\bar{r} = xi + yj + zk$.

$r^2 = x^2 + y^2 + z^2$

$r = |\bar{r}|$

diff w.r.t x	diff w.r.t y.	diff w.r.t z.
$2r\frac{\partial r}{\partial x} = 2x$.	$2r\frac{\partial r}{\partial y} = 2y$.	$2r\frac{\partial r}{\partial z} = 2z$.
$\boxed{\frac{\partial r}{\partial x} = \frac{x}{r}}$	$\boxed{\frac{\partial r}{\partial y} = \frac{y}{r}}$.	$\boxed{\frac{\partial r}{\partial z} = \frac{z}{r}}$.

LHS

$\Rightarrow$ Consider $\underline{\nabla f(r)}$.

$$\nabla f(r) = i\frac{\partial}{\partial x} f(r) + j\frac{\partial}{\partial y} f(r) + k\frac{\partial}{\partial z} f(r)$$

$$= i f'(r)\frac{\partial r}{\partial x} + j f'(r)\frac{\partial r}{\partial y} + k f'(r).\frac{\partial r}{\partial z}$$

sub the values

$$= if'(r)\left(\frac{x}{r}\right) + jf'(r)\left(\frac{y}{r}\right) + k f'(r)\left(\frac{z}{r}\right)$$

Take common

$$\frac{f'(r)}{r}\left[xi + yj + zk\right]$$

$$= \frac{f'(r)\,\bar{r}}{r} \Rightarrow \text{RHS}.$$

③ If $a = x+y+z$.

$b = x^2+y^2+z^2$.

$c = xy+yz+zx$.

P.T. $[\text{grad}\, a, \text{grad}\, b, \text{grad}\, c] = 0$.

Sol

i) $\text{grad}\, a = \nabla a = i\frac{\partial a}{\partial x} + j\frac{\partial a}{\partial y} + k \cdot \frac{\partial a}{\partial z}$.

$= i(1) + j(1) + k(1)$. look at LHS. & derivate.

$= i + j + k$.

ii). $\text{grad} = \nabla b = i\frac{\partial b}{\partial x} + j\frac{\partial b}{\partial y} + k \cdot \frac{\partial b}{\partial z}$

$= i(2x) + j \cdot (2y) + k \cdot (2z)$.

$= 2xi + 2yj + 2zk$.

iii) $\text{grad}\, c\ \nabla c = i\frac{\partial c}{\partial x} + \frac{\partial c}{\partial y} + \frac{\partial c}{\partial z}$.

$= i(y+z) + j(x+z) + k \cdot (x+y)$

iv) $[\text{grade}\, a\, \text{rade}\, b, \text{grade}\, c] = 0$.

$$\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2x & 2y & 2z \\ y+z & x+z & y+x \end{vmatrix}$$

$$1\,(2y)(y+x)_1 - 1\,(\cancel{2}xy + 2x^2\cancel{z} - 2yz - 2z^2)$$

$$-2xz - 2z^2$$

$$+ (2x^2 + 2xz - 2y^2 - 2yz.$$

$$= 0.$$

Hence proved

---

(2) $\bar{e} \cdot \nabla f = 0$ directional derivative "

$\bar{e} \cdot \nabla \phi = 0$ $\bar{e}$ = unit vector

$$\bar{e} = \frac{\bar{n}}{|\bar{n}|}.$$

① Find the directional derivative of $f(x,y,z) = xy^2 + yz^3$ at pt $(2,-1,1)$. in the direction of vector $i+2j+2k$.

$DD = \bar{e} \cdot \nabla f = 0$.

Sol. $\bar{e} \cdot \nabla f = DD$, given $\boxed{f = xy^2 + yz^3}$

To find $\bar{e} \cdot \nabla f$

so we have to find grad f & unit vector

$$\nabla f = i \frac{\partial f}{\partial x} + j \cdot \frac{\partial f}{\partial y} + k \cdot \frac{\partial f}{\partial z}$$

$$\text{grad } f = \nabla f = i \frac{\partial}{\partial x}(xy^2 + yz^3) + j \frac{\partial}{\partial y}(xy^2 + yz^3)$$

$$+ k \cdot \frac{\partial}{\partial z}(xy^2 + yz^2).$$

$$= iy^2 + j(2xy + 1(z^3)) + k \cdot 2yz$$

$$= iy^2 + 2xyj + z^3 + 2yzk.$$

$$\boxed{\nabla f = iy^2 + j(2yx + z^3) + k\, 3z^2 y}$$

The unit vector in the directio of $i+2j+2k$.

$\boxed{\text{let } \bar{n} = i+2j+2k.}$

$\boxed{\bar{e} = \frac{\bar{n}}{|\bar{n}|}}$

$\bar{e} = \frac{\bar{n}}{|\bar{n}|} = \frac{i+2j+2k}{\sqrt{(1)^2+(2)^2+(2)^2}} = \frac{i+2j+2k}{3}$

$\boxed{\bar{e} = \frac{1}{3}(i+2j+2k).}$

$\therefore DD = \bar{e} \cdot \nabla f$

$= \frac{1}{3}(i+2j+2k) \cdot \left(iy^2 + j(2yx+z^3) + k(3z^2y)\right)$

$(\nabla f \cdot e) = \frac{1}{3}\left[y^2 + 4yx + 2z^3 + 6z^2y\right]$

at point $(2,-1,1)$

$x=2, y=-1, z=1.$

$= \frac{1}{3}\left[(-1)^2 + 4(-1)(2) + 2(1)^3 + 6(1)^2(-1)\right]$

$\frac{1}{3}\left[1 - 8 + 2 - 6\right] \Rightarrow \frac{1}{3}\left[-11\right]$

$\therefore \boxed{D = \frac{-11}{3}.}$

Q. Find DD of $2xy+z^2$ at $(1,-1,3)$ in the direction of $i+2j+3k$.

Given : vector $\bar{n} = i+2j+3k$.

point $(1,-1,3)$

where $x=1,\ y=-1,\ z=3$.

$$f = 2xy+z^2.$$

$$\nabla f = i\cdot\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}.$$

$$\nabla f = i(2y) + j(2x) + k(2z).$$

$$\bar{e} = \frac{\bar{n}}{|\bar{n}|} = \frac{i+2j+3k}{\sqrt{(1)^2+(2)^2+(3)^2}} = \frac{1}{\sqrt{14}}(i+2j+3k).$$

$$\therefore DD = \nabla f\cdot\bar{e} = i(2y)+j(2x)+k(2z)\cdot\frac{1}{\sqrt{14}}(i+2j+3k)$$

$$= \frac{1}{\sqrt{14}}\left[2y+4x+6z\right].$$

$$= \frac{1}{\sqrt{14}}(2(-1)+4(1)+6(3))$$

$$= \frac{1}{\sqrt{14}}(20) \Rightarrow \frac{20}{\sqrt{14}}.$$

① Find the DD of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction of $(2i - j - 2k)$. Ans $\frac{37}{3}$

• Find the DD of $\phi = xy + yz + zx$ in the direction of vector $i + 2j + 2k$ at point $(1, 2, 0)$. $\frac{10}{3}$

• Find the DD of $xyz^2 + xz$ at $(1, 1, 1)$ in the direction of normal to $\boxed{\text{surface}}$ $3xy^2 + y = z$ at $(0, 1, 1)$.

Sol Surface is f

$\therefore f = \underline{3xy^2 + y - z = 0}$.

Unit normal at $(0, 1, 1)$.

$$\text{grade } f = \nabla f = i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}.$$

$$\cancel{\nabla f = i(3y^2) + j(6x) + k(1)}$$

$$\nabla f = i(3y^2) + j(6xy + 1) + k(-1).$$

$$\nabla f_{(0,1,1)} = i3 + j - k = \bar{n}.$$

to $(1,1,1)$

$$\bar{e} = \frac{\bar{n}}{|\bar{n}|} = \frac{3i + j - k}{\sqrt{(3)^2 + (1)^2 + (-1)^2}} = \frac{1}{\sqrt{11}}(3i + j - k)$$

$$\bar{e} = \frac{1}{\sqrt{11}}(3i + j - k).$$

$\therefore$ DD of ∇g

$$\text{grad } g = \nabla g = i(yz^2 + z) + j(xz^2) + k(2xyz + x).$$

$$\nabla g(1,1,1) = i2 + j + 3k.$$

$\therefore$ DD of the given function in the direction of $\bar{e}$ at $(1,1,1) = \nabla g \cdot \bar{e}$

$\therefore$ $\nabla g \cdot \bar{e}$

$$= (2i + j + 3k) \cdot \frac{1}{\sqrt{11}}(3i + j - k)$$

$$= \frac{1}{\sqrt{11}}(6 + 1 - 3) = \boxed{\frac{4}{\sqrt{11}}}$$

- Find the DD of the function $xy^2+yz^2+3x^2$ along the tangent to the curve $x=t, y=t^2, z=t^3$ in pt $(1,1,1)$

Sol

Let $f = xy^2+yz^2+3x^2$.

$\nabla f = i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\cdot\frac{\partial f}{\partial z}$.

$= i\cdot\frac{\partial}{\partial x}(xy^2+yz^2+3x^2) + j\cdot\frac{\partial}{\partial y}(xy^2+yz^2+3x^2) + k\cdot\frac{\partial}{\partial z}(xy^2+yz^2+3x^2)$.

$= i(y^2+z) + j(x+z^2) + k.(2zy+2zx)$.

$= i(y^2+z) + j(x(1)+1(z^2)+0)$.
$+k(0+y(1)+1(x^2))$.

$= i(y^2+z)+j(2yx+z^2)+k(2yz+x^2)$.

$\nabla f(1,1,1) = i(1+1)+j(2+1)+k(2+1)$.

$= 2i+3j+3k$.

$\bar{e} = \frac{\bar{n}}{|\bar{n}|}$

Let $\bar{r}$ be the position vector of curve.

$$\boxed{\bar{r} = xi + yj + zk}$$

unit vector

w.r.t $\frac{d\bar{r}}{dt}$ is the tangent to given cur.

$\therefore$ unit vector along the tangent.

Given $x = t, \; y = t^2, \; t^3 = z$

$\bar{r} = xi + yj + zk$.

$\bar{r} = ti + t^2 j + t^3 z$.

$\left(\frac{d\bar{r}}{dt}\right)_{(1,1,1)} = i + 2t + 3t^2 z$.

$= i + 2j + 3k$

$\therefore \bar{n} = i + 2j + 3k$.

$\bar{e} = \frac{\bar{n}}{|\bar{n}|} = \frac{i + 2j + 3k}{\sqrt{14}}$

$DD = \nabla \cdot \bar{e}$

$= \frac{18}{\sqrt{14}}(3 + 6 + 9) = \frac{17}{\sqrt{14}}$

Evaluate the angles b/w the normal to surface $xy = z^2$ at point $(4,1,2)$ & $(3,3,-3)$.

Sol. Given surface $f(x,y,z) = xy = z^2$.

n_1, n_2 be the normal to this surface at $(4,1,2)$ & $(3,3,-3)$

$$\text{grad } f = i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\cdot\frac{\partial f}{\partial z}$$

$$\nabla f = i(y) + jx + k(-2z).$$

$$\nabla f_{(4,1,2)} = \boxed{i + 4j - 4k = n_1}$$

$\overline{n_1} = \text{grad } n_1 =$

$n_2 = \text{grad } n_2 \text{ at } (3,3,3) = \boxed{3i + 3j + 6z = n_2}$

Let θ be the angle b/w 2 normals

$$\cos\theta = \frac{\overline{n_1}\cdot\overline{n_2}}{|\overline{n_1}|\,|\overline{n_2}|} = \frac{(i+4j-4k)\cdot(3i\,j+6z)}{\sqrt{(1^2+4^2+(-4)^2}\sqrt{3^2+3^2+6^2}}$$

$$= \frac{3+12-24}{\sqrt{33}\sqrt{34}} = \frac{-9}{\sqrt{33}\sqrt{54}} = \cos\theta.$$

Find the angle b/w the surfaces $x^2+y^2+z^2=a$

& $z=x^2+y^2-3$ at pt $(2,-1,2)$.

Sol Let $\phi_1 = x^2+y^2+z^2-9$.

$\nabla\phi_1 = \text{grad } \phi_1 = i2x+j2y+k2z$.

let $\phi_2 = \cancel{z-(x^2-y^2+z)}\ x^2+y^2-z-3=0$.

$\nabla\phi_2 = \text{grad } \phi_2 = i2x+j2y \cancel{+} -kz$.

let normal $n_1 = (\nabla\phi_1)$ at $(2,-1,2)$ is.

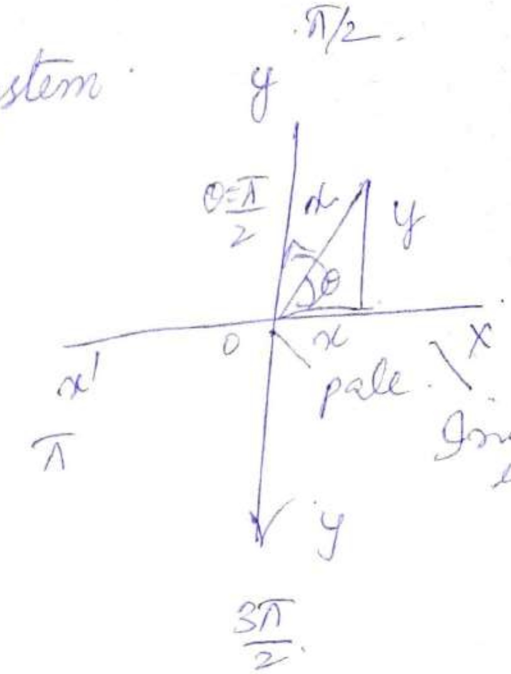
$i2x+j2y-kz = 4i-2j-2k = n_2$.

$\cancel{\cos\theta}$ $\dfrac{\overline{n_1}\cdot\overline{n_2}}{|\overline{n_1}|\cdot|\overline{n_2}|} = \dfrac{(4i-2j+4k)\cdot(4i-2j-2k)}{\sqrt{4^2}}$

Unit 3

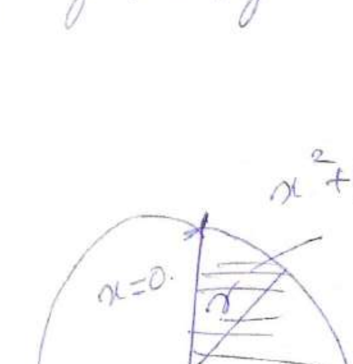
Polar Co-ordinate System:

$\cos\theta = \frac{x}{r}$



lowest angle $=0$.

1) Evaluate the integral by transforming into polar co-ordinate $\int_0^a \int_0^{\sqrt{a^2-x^2}} y\sqrt{x^2+y^2}\,dx\,dy$.



Sol x limits 0 to a

y limits 0 to $\sqrt{a^2-x^2}$

$y=0$ & $y=\sqrt{a^2-x^2}$.

SOBS.

$\Rightarrow y^2 = a^2-x^2$

$x^2+y^2 = a^2$.

$x=r\cos\theta$
$y=r\sin\theta$

$\boxed{r=a}$

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

$dx\,dy = r\,dr\,d\theta$

r limits in terms of radius

$r = 0$ to a

θ limits $\theta = 0$ to $\pi/2$.

$$\int_{r=0}^{a}\int_{\theta=0}^{\pi/2} r\sin\theta\sqrt{(r^2\cos^2\theta + r^2\sin^2\theta)}\; r\,dr\,d\theta$$

$$= \int_{r=0}^{a}\int_{\theta=0}^{\pi/2} r\sin\theta\sqrt{r^2(\sin^2\theta + \cos^2\theta)}\; r\,dr\,d\theta$$

$$= \int_{r=0}^{a}\int_{\theta=0}^{\pi/2} r\sin\theta\; r\,.\,r\,dr\,d\theta$$

$$= \int_{r=0}^{a}\int_{\theta=0}^{\pi/2} r^3\sin\theta\,dr\,d\theta$$

Integrate w.r.t θ.

$$\int_{r=0}^{a} r^3\left[-\cos\theta\right]_0^{\pi/2} dr$$

$$\int_{r=0}^{a} r^3\left[-\cos\pi/2 + \cos(0)\right] dr$$

$$\int_0^a r^3[0+1]\,dr$$

$$= \left[\frac{r^4}{4}\right]_0^a \Rightarrow \frac{a^4}{4} \,//$$

- Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2+y^2)\,dy\,dx$ by changing into polar co-ordinates.

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2+y^2)\,dy\,dx.$$

x limits $= 0$ to 2

y limits $= 0$ to $\sqrt{2x-x^2}$

$y = 0$

$y = \sqrt{2x-x^2}$.

S.O.B.S

$y^2 = 2x - x^2$.

$2x - x^2 = y^2$

$-x^2 - y^2 = -2x$.

$$\boxed{x^2+y^2 = 2x}$$

w.k.t $x^2+y^2=r^2$.

w.k.t $x = r\cos\theta$

$y = r\sin\theta$.

$\boxed{y^2+x^2 = 2x}$

$r^2 = 2r\cos\theta$

$\boxed{r = 2\cos\theta}$

Initial limits

$x=0,\ y=0$.

$x = r\cos\theta\ 0$.

$y = r\sin 0$.

$\boxed{\theta = \frac{\pi}{2}}$

$$= \int_0^{\pi/2}\int_0^{2\cos\theta} r^2\, r\, dr\, d\theta$$

$$\int_0^{\pi/2}\int_0^{2\cos\theta} r^3\, dr\, d\theta$$

$$\int_0^{\pi/2}\left[\frac{r^4}{4}\right]_0^{2\cos\theta} d\theta$$

subs $2\cos\theta$ in r.

$$\int_0^{\pi/2}\left[\frac{(2\cos\theta)^4}{4}\right]_0^{2\cos\theta} d\theta$$

$$\int_0^{\pi/4}\frac{2^4}{4}\left[\cos^4\theta\, d\theta\right]$$

$$\frac{2^4}{4}\left[4\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\frac{\pi}{2}\right] = \frac{3\pi}{2}.$$

Evaluate $\boxed{\iint r\sin\theta\, dr\, d\theta}$ over the cardioid $r = a(1-\cos\theta)$ above the Initial line

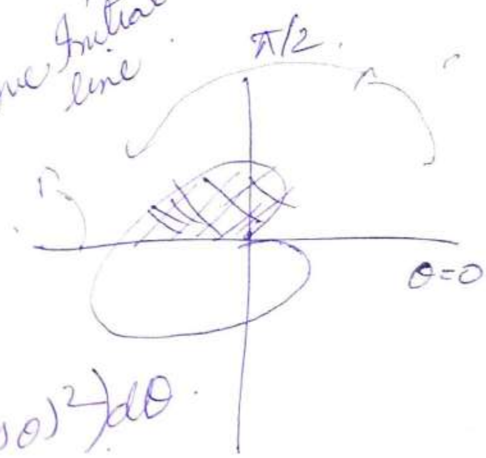
already in r & θ form.

$r = a(1-\cos\theta)$

$r = a(1-\cos\theta)$

Sol limits of $\theta = 0$ to $\pi/2$.

limits of $r = 0$ to $a(1-\cos\theta)$.



$$\int_{\theta=0}^{\pi}\int_{r=0}^{a(1-\cos\theta)} r\sin\theta\, dr\, d\theta.$$

$$\int_{\theta=0}^{\pi}\left[\frac{r^2}{2}\right]_0^{1-\cos\theta}\sin\theta\, d\theta = \frac{1}{2}\int_{\theta=0}^{\pi}\sin\theta\left[a^2(1-\cos\theta)^2\right]d\theta.$$

$$= \frac{a^2}{2}\int_{\theta=0}^{\pi}\sin\theta(1-\cos\theta)^2\, d\theta.$$

$$\boxed{\therefore \int (f(x))^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+c} + c.}$$

$$\frac{a^2}{2}\left[\frac{(1-\cos\theta)^{2+1}}{2+1}\right]_0^{\pi}.$$

$$=\frac{a^2}{2}\left[(1-\cos\pi)^3-(1-\cos 0)^3\right].$$

$$\frac{a^2}{6}\left[8\right]=\frac{4a^2}{3}$$

Change the order of Integration.

① Swap limits
② check the strip
③ y - limits → const. } vertical
x - limits → variables. } ⇓ horizontal

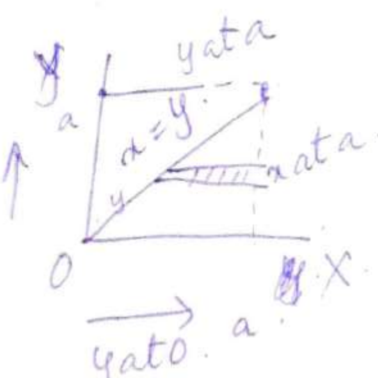
① Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2}\,dx\,dy$ by
changing the order of Integration.

y unte adi x limit

$$\int_{y=0}^{y=a} \int_{x=y}^{x=a} \frac{x}{x^2+y^2}\,dx\,dy.$$

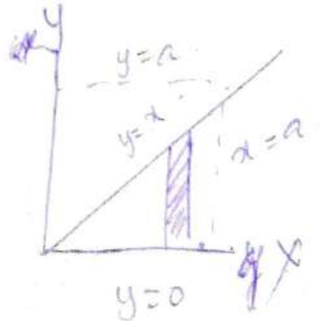
variables.
x limits : $x = y$ to $x = a$
y limits : $y = 0$ to a.
constants.

old



parallel to x-axis.

New



parallel to y-axis

y limits : $y = 0$ to x.
x limits : 0 to a

Me conti

Divergent of a vector :- Let 'f' be any continuously differentiable vector point function Then $i \cdot \frac{\partial f}{\partial x} + j \cdot \frac{\partial f}{\partial y} + k \cdot \frac{\partial f}{\partial z}$ is called the divergent of $\bar{f}$ i.e $\text{div} \bar{f} = \boxed{i \cdot \frac{\partial f}{\partial x} + j \cdot \frac{\partial f}{\partial y} + k \cdot \frac{\partial f}{\partial z}}$

Solenoidal Vector :- A vector point function 'f' is said to be solenoidal if $\boxed{\text{div} \bar{f} = 0}$

① If $f = xy^2 i + 2x^2 yz j - 3yz^2 k$ find div f at $(1,-1,1)$

sol

$f = xy^2 i + 2x^2 yz j - 3yz^2 k.$

$\text{div} f = i(xy^2) + j(2x^2yz) - k(3yz^2)$

$= \frac{\partial}{\partial x}(xy^2) + \frac{\partial}{\partial y}(2x^2yz) - \frac{\partial}{\partial z}(3yz^2)$

$\boxed{\text{div} f = y^2 + 2x^2 z - 6yz}$

at $(1,-1,1)$.

$(-1)^2 + 2(1)^2(1) - 6(-1)(1)$

$= 1 + 2 + 6 = 9$

② If $\bar{f} = (x+3y)i + (y-2z)j + (x+Pz)k$ is solenoidal, find P.

$\bar{f} = i\cdot(x+3y) + j\cdot(y-2z) + k\cdot(x+Pz)$

$\frac{\partial}{\partial x}(x+3y) + \frac{\partial}{\partial y}(y-2z) + \frac{\partial}{\partial z}(x+Pz)$.

div f = ~~3y - 2z + x + P~~

$1+1+P$

div f = $2+P$.

$\nabla\cdot f = P+2$.

But it is solenoidal

div f = 0,

then, $P+2=0$.

$\boxed{P=-2}$

~~**~~ find div $\bar{f}$ where $\bar{f} = r^n\bar{r}$. Find n if its solenoidal.

(OR).

Prove $r^n\bar{r}$ is solenoidal if $n=3$.

(OR)

Prove div $(r^n\bar{r}) = (n+3)r^n$. Hence show $\bar{r}/r^3$ is solenoidal given $f = r^n \bar{r}$.

$\bar{r} = xi + yj + zk$.

$r = |\bar{r}|$

and w.k.t. $r^2 = x^2 + y^2 + z^2$ — ① — ②

diff ② partially w.r.t 'x'

$2r\dfrac{\partial r}{\partial x} = 2x$ | $2r\dfrac{\partial r}{\partial y} = 2y$ | $2r\dfrac{\partial r}{\partial z} = 2z$

$\boxed{\dfrac{\partial r}{\partial x} = \dfrac{x}{r}}$ | $\boxed{\dfrac{\partial r}{\partial y} = \dfrac{y}{r}}$ | $\boxed{\dfrac{\partial r}{\partial z} = \dfrac{z}{r}}$

$f = r^n \bar{r} = r^n(xi + yj + zk)$.

$\operatorname{div} f = i \cdot \dfrac{\partial f}{\partial x} + j \cdot \dfrac{\partial f}{\partial y} + k \cdot \dfrac{\partial f}{\partial z}$.

$\dfrac{\partial}{\partial x}(x r^n) + \dfrac{\partial}{\partial y}(y r^n) + \dfrac{\partial}{\partial z}(z r^n)$.

$\boxed{\dfrac{\partial}{\partial x}(uv) = uv' + vu'}$

$\cancel{= x n r^{n-1} + r^n + y n r^{n-1} + r^n + z n r^{n-1} + r^n}$

$3r^n + n r^{n-1}[x + y + z]$.

$$\text{div} f = \frac{\partial}{\partial x}(x r^n) + \frac{\partial}{\partial y}(y r^n) + \frac{\partial}{\partial z}(z r^n).$$

$$\Rightarrow x n r^{n-1}\left(\frac{\partial r}{\partial x}\right) + r^n \quad + .$$

$$y n r^{n-1}\left(\frac{\partial r}{\partial y}\right) + r^n \quad + .$$

$$z n r^{n-1}\left(\frac{\partial r}{\partial z}\right) + r^n$$

$$\Rightarrow 3r^n + x n r^{n-1}\left(\frac{x}{r}\right) + y n r^{n-1}\left(\frac{y}{r}\right) + z n r^{n-1}\left(\frac{z}{r}\right)$$

$$= 3r^n + n r^{n-1}\left(\frac{x^2}{r}\right) + n r^{n-1}\left(\frac{y^2}{r}\right) + n r^{n-1}\left(\frac{z^2}{r}\right)$$

$$3r^n + n r^{n-1}\left[\frac{x^2}{r} + \frac{y^2}{r} + \frac{z^2}{r}\right].$$

$$3r^n + n r^{n-1}\left[\frac{r^{2,1}}{\cancel{r}}\right].$$

$$3r^n + n r^{n-2}$$

$$\boxed{(n+3) r^n.}$$

solenoidal means $\text{div} f = 0$.

$\text{ans} = 0$

$$(n+3) r^n = 0$$

$$n + 3 = 0$$

$$\boxed{n = -3}$$

**

Evaluate $\nabla \cdot \left(\frac{\bar{r}}{r^3}\right)$ where $\bar{r} = xi + yj + zk$.

Ie $r = |\bar{r}|$ (OR) Show that $\frac{\bar{r}}{r^3}$ is solenoidal

Sol $\bar{r} = x\bar{i} + yj + zk$; $r = |\bar{r}|$. From r^2.

$r^2 = x^2 + y^2 + z^2 \Rightarrow \boxed{\frac{\partial r}{\partial x} = \frac{x}{r}}, \boxed{\frac{\partial r}{\partial y} = \frac{y}{r}}, \boxed{\frac{\partial r}{\partial z} = \frac{z}{r}}$

$f = \bar{r}/r^3$

$\cancel{\text{div } f} = i \cdot \cancel{\frac{\partial f}{\partial x}} + j \cancel{\frac{\partial f}{\partial y}} + k \cancel{\frac{\partial f}{\partial z}}$.

$\cancel{\frac{\partial}{\partial x}} + \cancel{\frac{\partial}{\partial y}} + \cancel{\frac{\partial}{\partial y}}$

$f = \bar{r} \cdot r^{-3}$

$= (xi + yj + zk) r^{-3}$

$= i x r^{-3} + j y r^{-3} + k z r^{-3}$

now it is in UV format.

~~$3r^{-3}[ix + jy + kz]$~~

$\text{div } f = i \cdot \frac{\partial f}{\partial x} + j \cdot \frac{\partial f}{\partial y} + k \cdot \frac{\partial f}{\partial z}$.

$\frac{\partial}{\partial x}(x r^{-3}) + \frac{\partial}{\partial y}(y r^{-3}) + \frac{\partial}{\partial z}(z r^{-3})$

$x(-3) r^{-3-1} \frac{\partial r}{\partial x} + r^{-3} + y(-3) r^{-3-1} \frac{\partial r}{\partial y} + r^{-3}$

$+ z(-3) r^{-3-1} \frac{\partial r}{\partial z} + r^{-3}$

$$-3x r^{-4}\frac{\partial r}{\partial x}+r^{-3}$$

$$3r^{-3}-3xr^{-4}\left(\frac{x}{r}\right)-3yr^{-4}\left(\frac{y}{r}\right)-3zr^{-4}\left(\frac{z}{r}\right)$$

$$3r^{-3}-3x^2r^{-5}-3y^2r^{-5}-3z^2r^{-5}.$$

$$3r^{-3}-3r^{-5}(x^2+y^2+z^2).$$

$$=3r^{-3}-3r^{-5}(r^2).$$

$$=3r^{-3}-3r^{-3}$$

$$=0.$$

$\therefore \frac{\bar{r}}{r^3}$ is solenoidal.

M2

Condition of Exactness.

$$\boxed{Mdx + Ndy = 0}$$

be exact is

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Steps:

1) $Mdx + Ndy = 0$
2) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, check exactness = ?
3) Final Solution is given by $\boxed{C = ?}$

$$\underset{y = \text{cont}}{\int Mdx} + \underset{\substack{\text{terms free}\\ \text{from } x}}{\int Ndy} = C$$

OR

$$\underset{x = \text{cont}}{\int Ndy} + \underset{\text{terms free from } y}{\int Mdx} = C$$

① Solve.

$$\left[\log(x^2+y^2)+\frac{2x^2}{x^2+y^2}\right]dx+\left[\frac{2xy}{x^2+y^2}\right]dy=0.$$

Step 1 = $\boxed{Mdx+Ndy=0}$

Given :-

$$\left[\log(x^2+y^2)+\frac{2x^2}{x^2+y^2}\right]dx+\left[\frac{2xy}{x^2+y^2}\right]dy=0. \quad \text{—①}$$

compare eqn① with $Mdx+Ndy=0$.

$$M=\log(x^2+y^2)+\frac{2x^2}{x^2+y^2}.$$

$$N=\frac{2xy}{x^2+y^2}$$

Step 2 :- $\boxed{\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}}$ Check exactness.

$$\frac{\partial M}{\partial y}=\frac{\partial}{\partial y}\left[\log(x^2+y^2)+\frac{2x^2}{x^2+y^2}\right]$$

$\boxed{x \text{ wrt } y=0.}$

$$=\frac{1}{x^2+y^2}\times(0\times 2y)+(2x^2)$$

$\boxed{\log x=\frac{1}{x}}$

$$\times\frac{-1}{(x^2+y^2)^2}\times(0+2y).$$

$$\frac{\partial M}{\partial y} = \frac{2y}{x^2+y^2} + \frac{(-4x^2y)}{(x^2+y^2)^2}$$

$$\frac{\partial N}{\partial y} = \frac{\partial}{\partial x}\left(\frac{2xy}{x^2+y^2}\right] \quad \text{—} \ (2)$$

$$= \frac{(x^2+y^2)\cdot\frac{\partial}{\partial x}(2xy) - (2xy)\frac{\partial}{\partial x}(x^2+y^2)}{(x^2+y^2)}$$

Linear form.

Form 1

$$\boxed{\frac{dy}{dx} + Py = Q}$$ – linear in y.

where P & Q are F^n of x only.

$$\boxed{IF = e^{\int P dx}}$$

GS is.

$$\boxed{y(IF) = \int Q(IF)\,dx + c}$$

Note.

(1) power of y must be 1.

(2) coeff of $\frac{dy}{dx} = 1$.

Form 2

$$\boxed{\frac{dx}{dy} + Px = Q}$$ – linear in x.

where P & Q are F^n of y only.

$$\boxed{IF = e^{\int P dy}}$$ $GS = x(IF) = \int Q(IF)\,dy + c$.

Note:

power of x must be 1.

power of $\frac{dx}{dy}$ m be 1.

① $\cdot\ (\not{x}\ e$

① Solve $(1+x^2)\dfrac{dy}{dx} + 2xy = 4x^2,\ y(0)=0.$

$$\frac{dy}{dx} + \frac{2x}{(1+x^2)}y = \frac{4x^2}{1+x^2}.$$

$$IF = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1+x^2.$$

$$GS = \boxed{y(IF) = \int Q(IF)\,dx + c.}$$

$$y(IF) = \int \frac{4x^2}{1+x^2}\cancel{(IF)}\,dx + C.$$

$$y(1+x^2) = \int 4x^2\,dx + C$$

$$= 4\frac{x^3}{3} + C.$$

$$y(1+x^2) = \frac{4}{3}x^3 + C$$

at $x=0,\ y=0 \Rightarrow C=0$

$$y(1+x^2) = \frac{4}{3}x^3.$$

② If 30% of a radioactive substance disappeared in 10 days, how long will it take

Let M be amt of radioactive substance present at any time t.

$$\frac{dM}{dt} \propto M$$

$$\frac{dM}{dt} = kM$$

$$\frac{dM}{M} = -kdt$$

$$\therefore \log M = -kt + A$$

Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$.

$M dx + N dy = 0$.

$(y\cos x + \sin y + y)\, dx + (\sin x + x\cos y + x)\, dy = 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

$\frac{\partial M}{\partial y} = y\cos x + \sin y + y$

$= \cos x + \cos y + 1$.

$\frac{\partial N}{\partial x} = \sin x + x\cos y + x$.

$= \cos x + \cos y + 1$.

$\therefore GS = \int M dx =$

$\int M dx = \int (y\cos x + \sin y + y)\, dx$.

$= (y\sin x + x\sin y + xy)$.

all have x.

$\int$ (Terms free from x).

$= \int 0\, dy = 0$.

$\therefore GS = \int M dx + \int (\text{Terms free from } x)\, dy = c$.

$\therefore y\sin x + x\sin y + xy + 0 = c$.

Solve : $2(1+x^2\sqrt{y})y\,dx + (x^2\sqrt{y}+2)x\,dy = 0$.

comparing with $M\,dx + N\,dy = 0$.

$\therefore M = 2(1+x^2\sqrt{y})y$.

$\therefore N = (x^2\sqrt{y}+2)x$.

$M = 2(1+x^2\sqrt{y})y$.

$= 2y(1+x^2\sqrt{y})$.

$2y + 2x^2y\sqrt{y}$.

$N = (x^2\sqrt{y}+2)x$.

$= x(x^2\sqrt{y}+2)$.

$= x^3\sqrt{y}+2x$.

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

$$\therefore \frac{\partial M}{\partial y} = 2y + 2x^2\underline{y\sqrt{y}}$$

$$= 2 + \cancel{2}x^2 \cdot \frac{3}{\cancel{2}} y^{\frac{1}{2}}$$

$$= 2 + 3x^2 y^{\frac{1}{2}}$$

$$= 3x^2\sqrt{y}+2.$$

$N = \frac{\partial N}{\partial x} = x^3\sqrt{y}+2x$

$= 3x^2\sqrt{y}+2$.

$\boxed{y\sqrt{y} = \frac{3}{2}y^{1/2}}$

$y\sqrt{y} = y^1 \times y^{\frac{1}{2}}$

$= y^{\frac{3}{2}}$.

$\therefore$ Given diff eqn is exact.

$\int M\,dx = \cancel{2yx + \frac{2x^3}{3}} \cdot \frac{2}{3}\left(x^3 y^{\frac{3}{2}}\right) + 2xy = c$.

(Q). $2xy + y - \tan y - dx + x^2 - x\tan^2 y + \sec^2 y\, dy.$

(1) $M dx + N dy = 0.$

$M = 2xy + y - \tan y.$

$\frac{\partial M}{\partial y} = 2x + \frac{1}{\cancel{x}} - \sec^2 \cancel{xy}..$

$= 2x - \sec^2 y + 1.$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(2xy + y - \tan y).$

$= 2x(1) + 1 - \sec^2 xy$

$2x - \sec^2 xy + 1.$

$2x - \tan^2 y$

$N = x^2 - x\tan^2 y + \sec^2 y.$

$\frac{\partial N}{\partial x} = 2x - (1)(\tan^2 y + \cancel{y} \sec^2 y \tan^2 y$

$= 2x - \tan^2 y.$

$\frac{\partial N}{\partial x} = 2x - \tan^2 y.$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}. \quad \neq.$

(Q). $(y \sin 2x)dx - (y^2 + \cos^2 x)\, dy = 0.$

$M dx + N dy = 0$

$M = y \sin 2x.$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(y \sin 2x).$

$= \cancel{\cos 2x} \sin 2x$

$N = -(y^2 + \cos^2 x)$

$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x} -(y^2 + \cos^2 x)$ 0

$= 2y + \sin^2 x.$

$GS = \int M\,dx + \int N\,dy = C.$

$y = \text{const}$, free from x terms.

$N = -y^2 + \cancel{\cos 2x}$.

$N = -y^2$.

$\int (y \sin 2x)\,dx + \int -y^2\,dy = c.$

$y \int dx + \int \sin 2x\,dx - \frac{y^3}{3} = c.$

$y - \frac{\cos 2x}{2} - \frac{y^3}{3} = c.$

$\frac{-y\cos 2x}{2} - \frac{y^3}{3} = c.$

$-3y\cos 2x - 2y^3 = 6c.$

$-[3y\cos 2x + 2y^3] = 6c.$

$\therefore 3y\cos 2x + 2y^3 = -6c.$

Q. $x(1+y^2)\,dx + y(1+x^2)\,dy = 0.$

$M\,dx + N\,dy = 0.$

$\cancel{x(1+y^2)}$

$M = x(1+y^2)$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left(x(1+y)^2\right).$

$2xy.$

$N = y(1+x^2).$

$\frac{\partial N}{\partial x} = y(2x).$

$= 2xy$

$GS = \int M dx + \int N dy = c$

$x + xy^2 + y + y x^2$

$x + xy^2 + y$

$\int \cancel{x} +$

$\int x\, dx + \int xy^2 dx + \int y \cdot dy = c$

$\int x\, dx + y^2 \int x\, dx + \int y\, dy = c$

$\frac{x^2}{2} + \left(y^2 \cdot \frac{x^2}{2}\right) + \frac{y^2}{2} = c$

$\therefore x^2 + y^2 x^2 + y^2 = 2c$

Q. $(xy \cos xy + \sin xy)\, dx + (x^2 \cos xy) dy = 0$

$M dx + N dy = 0$

$M = xy \cos xy + \sin xy$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(xy \cos xy + \sin xy)$

= ~~bc~~ Apply UV formulae

$[\because UV = UV' + VU']$

$= x \cdot \frac{\partial}{\partial y}(y \cos xy) + \frac{\partial}{\partial y}(\sin xy)$

$\cancel{N = x^2 \cos xy}$

$x[y(-\sin xy)x + \cos xy(1)]$

$+ \cos xy$

$= x[-xy$

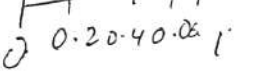
Eulers Method

- $y_1 = y_0 + h\, f(x_0, y_0)$.

- $\boxed{y_n = y_{n-1} + h\, f(x_{n-1}, y_{n-1})}$

① using Eulers method, find approx value of y at $x=1$ in 5 steps. taking, $h = 0.2$ given

$\frac{dy}{dx} = x + y$. Ge $y(0) = 1$.



$\frac{dy}{dx} = f(x,y) = x + y$

$f(x_0, y_0) = x_0 + y_0 = 0 + 1 = 1$.

at $x_1 = 0.2$, $y_1 = y_0 + h f(x_0, y_0)$.

$= 1 + (0.2)(1)$.

$\boxed{y_1 = 1.2}$

$\therefore f(x_1, y_1) = x_1 + y_1 = 0.2 + 1.2 = 1.4$.

At $x_2 = 0.4$. $= y_2 = y_1 + h f(x_1, y_1)$.

$= 1.2 + (0.2)(1.4)$.

$\boxed{y_2 = 1.48}$

$\therefore f(x_2, y_2) = 0.4 + 1.48 = 1.88$

at $x_3 = 0.6$.

$$y_3 = y_2 + hf(x_2, y_2).$$

$$1.48 + (0.2)(1.88).$$

$$\boxed{y_3 = 1.856}.$$

$$x_3 + y_3 = 0.6 + 1.856 = 2.456.$$

$f(x_3, y_3) =$

at $x_4 = 0.8$.

$$y_4 = y_3 + hf(x_3, y_3).$$

$$= 1.856 + (0.2)(2.456).$$

$$\boxed{y_4 = 2.3472}.$$

$$f(x_4, y_4) = x_4 + y_4 = 0.8 + 2.3472 = 3.1472.$$

At $x_5 = 1$, $\quad y_5 = y_4 + hf(x_4, y_4).$

$$= 2.3472 + 0.2(3.1472).$$

$$\boxed{y_5 = 2.97664}.$$

~~[illegible]~~

* LDE with constant co-effuent.

$$\boxed{\frac{dy}{dx} + Py = Q}$$

* Operator D:

$$\frac{dy}{dx} = \frac{d}{dx}(y) = Dy.$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = D(Dy) = D^2y.$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = D^3y.$$

$$\boxed{\frac{d^ny}{dx^3} = D^ny}$$

General Solution

Complimentary function

$\downarrow$

Particular Integral

$\downarrow$

GS = CF + PI

CF
↓
types (4).

PI
↓
Types (6).

① Calu.
② Types ch !!

* Auxillary eqn.

① Solve: $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$.

∴ The auxillary eqn is,

$D^3y - 6D^2y + 11Dy - 6y = 0$.

∴ $y(D^3 - 6D^2 + 11D - 6) = 0$.

∴ $D^3 - 6D^2 + 11D - 6 = 0$.

∴ $D = 1, 2, 3$ ... roots of LDE.

Mode.
5, EQN
4, $ax^3 + bx^2 + cx + d = 0$
Value =.

② Solve $\frac{d^3y}{dx^3} - \frac{5d^2y}{dx^2} + 8\frac{dy}{dx} - 4y = 0$.

The auxillary eqn is

$D^3y - 5D^2y + 8Dy - 4y = 0$.

$y(D^3 - 5D^2 + 8D - 4) = 0$.

$D^3 - 5D^2 + 8D - 4 = 0$.

1, 2, 2 .... roots.

1, 2 in calu
repeted root
∴ $x_3 = 2$.

Complementary function

- Roots r. non repeating
- Roots r. repeating
- Complex non repeting roots
- Complex repeating roots

Type 1

Real & non repeating roots.

$$\boxed{y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x} + \cdots}$$

where m_1, m_2, m_3 are roots of eqn.

① Solve $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = 0$.

$$D^3y + 2D^2y - 5Dy - 6y = 0$$

$$D^3 + 2D^2 - 5D - 6 = 0$$

$$\therefore D = \underbrace{2, -1, -3}$$

roots are not repeating here.

$\therefore$ use type 1.

$\therefore$ CF is

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$$

$$= c_1 e^{-x} + c_2 e^{2x} + c_3 e^{-3x}$$

(2) Type (2)

Real repeating 2.

- $y = (C_1 + C_2 x) e^{mx}$ .... if it is repeating twice.
- $y = (C_1 + C_2 x + C_3 x^2) e^{mx}$ .... 3 times.

(1). $\frac{d^3y}{dx^3} - \frac{3d^2y}{dx^2} + 4y = 0$

$D^3 y - 3 D^2 y + 4y = 0$.

$y(D^3 - 3D^2 + 4) = 0$.

$D^3 - 3D^2 + 4 = 0$.

$D = -1, 2, 2$.

CF=

$(C_1 + C_2 x) e^{mx}$

~~$= (C_1 + C_2 x) e$~~

for -1 root use type 1.

for 2, 2 use type 2

$y = C_1 e^{-mx} + (C_2 + C_3 x) e^{mx}$.

$$\boxed{y = C_1 e^{-1x} + (C_2 + C_3 x) e^{2x}}.$$

(1) $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0.$

(2) $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} = 0$

(3). $\frac{d^3y}{dx^3} + 4.\frac{d^2y}{dx^2} + \frac{dy}{d^2} - 6y = 0$

(1). $D^3y - 3D^2y + 3Dy - y = 0.$

$y(D^3 - 3D^2 + 3D - 1) = 0.$

$1, 1, 1.$

$y = (C_1 + C_2x + C_3x^2).e^{mx}.$

$(C_1 + C_2x + C_3x^2)e^{x}.$

(3) $D^3 + 4D^2 + 1D - 6 = 0.$

$1, -2, -3$

$C_1e^{mx} + C_2e^{mx} + C_3e^{mx}$

$C_1e^{x} + C_2e^{-2x} + C_3e^{-3x}$

(2). $D^3y + 2D^2y + 0 + 0.$

$y(D^3 + 2D^2 + 0 + 0).$

$-2, 0, 0.$

$C_1e^{mx} + (C_1 + C_2x).e^{mx}.$

$C_1e^{-2x} + (C_2 + C_3x).e^{0x}.$

Type 3.

complex with non repeating roots.

$$\boxed{y = e^{\alpha x}\left[c_1 \cos\beta x + c_2 \sin\beta x\right]}$$

where α = real part, β is img part.

(1). Solve $\frac{d^3y}{dx^3} + y = 0$.

$\therefore D^3y + y = 0$.

$y(D^3 + 1) = 0$.

$D^3 + 0 + 0 + 1 = 0$.

$y = -1,\ \underbrace{\frac{1}{2} \pm \frac{\sqrt{3}}{2} i}$

$y = e^{\alpha x}\left[c_1 \cos\beta x + c_2 \sin\beta x\right)$.

$y = c_1 e^{-1x} + e^{\frac{1}{2}x}\left[c_2 \cos\frac{\sqrt{3}}{2}x + c_3 \sin\frac{\sqrt{3}}{2}x\right]$.

(2). Solve $\frac{d^3y}{dx^3} + 8y = 0$.

$D^3y + 0 + 0 + 8y = 0$.

$y(D^3 + 0 + 0 + 8) = 0$.

$-2,\ 1 + \sqrt{3}i,\ 1 - \sqrt{3}i$. $\quad i$ ✗

$c_1 e^{-2x} + e^{1x}\left[c_2 \cos\sqrt{3}\,x + c_3 \sin\sqrt{3}\,x\right]$. $\quad x$ ✓

Type 4.

complex repeating roots. (highest power 4).

$y = e^{\alpha x} \left[(c_1 + c_2 x) \cos \beta x + (c_3 + c_4 x) \sin \beta x \right]$.

where, α = real part, β = Imaginary part.

(1) Solve $\dfrac{d^4y}{dx^4} + 6\dfrac{d^2y}{dx^2} + 9y = 0$.

$D^4y + 6D^2y + 9y = 0$.

$(D^4 + 6D^2 + 9) = 0$. $\longrightarrow$ check if it is perfect sq

$(D^2+3)(D^2+3) = 0$.

$D^2 = -3, -3$.

$D = \pm\sqrt{3}i, \pm\sqrt{3}i$

CF is

$y = e^{0x} \left[(c_1 + c_2 x) \cos \sqrt{3}x + (c_3 + c_4 x) \sqrt{3}x \right.$

Particular Integral.

(RHS has some func).
If RHS = 0 find CF.

* when $X = e^{ax} \Rightarrow$ RHS.

$$\frac{1}{f(D)} \cdot e^{ax} = \frac{1}{f(a)} \cdot e^{ax}.$$

① Solve : $(D^3 - 2D^2 - 5D + 6)y = e^{3x} + 8$.

for CF, Auxillary eqn is

$$(D^3 - 2D^2 - 5D + 6)y = 0.$$

$$D^3 - 2D^2 - 5D + 6 = 0.$$

$$y = -2, 3, 1.$$

$$\therefore CF = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}.$$

$$= c_1 e^{x} + c_2 e^{-2x} + c_3 e^{3x}.$$

for PI.:

$$(D^3 - 2D^2 - 5D + 6)y = e^{3x} + 8.$$

8(1).

$$y = \frac{1}{D^3 - 2D^2 - 5D + 6} \cdot e^{3x} + 8.$$

$$= \frac{1}{(D^3 - 2D^2 - 5D + 6)} \cdot e^{3x} + 8e^{0x} \cdot \frac{1}{D^3 - 2D^2 - 5D + 6}.$$

$$\frac{1}{3^3-2(3)^2-5(3)+6}\cdot e^{3x}+\frac{1}{0-2(0)-5(0)+6}\cdot 8e^{0x}$$

$e^{(3)x}$ sub as D.

$$\frac{1}{0}\cdot e^{3x}+\frac{1}{6}\cdot 8e^{0x}$$

Note ÷ when PI is not defined then,

$$\frac{1}{f(D)}\cdot e^{ax}=x\cdot\frac{1}{f'(D)}\cdot e^{ax}$$

$$\therefore x\cdot\frac{1}{f'(a)}\cdot e^{ax}$$

Dif

$$\therefore y=\frac{1}{(D^3-2D^2-5D+6)}e^{3x}+\frac{8}{(D^3-2D^2-5D+6)}\cdot e^{0x}$$

Jisme 0 ara usi me diff karo

$$\frac{x}{3D^2-4D-5}\cdot e^{3x}+\frac{8}{D^3-2D^2-5D+6}\cdot e^{0x}$$

now add 3 in D.

$$=\frac{x}{10}\cdot e^{3x}+\frac{8}{6}\cdot e^{0x}$$

$$\frac{x\cdot e^{3x}}{10}+\frac{4}{3}$$

$$GS = CF + PI.$$

$$y = \left[c_1 e^{x} + c_2 e^{-2x} + c_3 e^{3x}\right] + \left[\frac{x}{10} e^{3x} + \frac{4}{3}\right].$$

② Solve $\dfrac{d^3y}{dx^3} - 4\dfrac{dy}{dx} = 2\cosh^2 2x$.

$$\cosh x = \frac{e^{x} + e^{-x}}{2}.$$

CF = Auxilary.

$$(D^3y - 4Dy) = 0.$$

$$(D^3 - 4D)y = 0.$$

$$D^3 - 4D = 0.$$

$$D(D^2 - 4D) = 0.$$

$$D = 0, 2, -2.$$

$$\therefore \cancel{CF = c_1 e^{0x} + (c_2 + c_3) e}$$

$$CF = c_1 e^{0x} + c_2 e^{2x} + c_3 e^{-2x}.$$

$$= \boxed{c_1 + c_2 e^{2x} + c_3 e^{-2x}}.$$

$$PI = \frac{1}{(D^3 - 4D)} \cdot 2\cosh^{2} 2x.$$

$$\boxed{a^2 + b^2}.$$

$$= \frac{1}{D^3 - 4D} \cdot 2\left(\frac{e^{2x} + e^{-2x}}{2}\right)^{2}$$

$$= \frac{1}{D^3 - 4D} \cdot 2\left(\frac{e^{4x} + 2 \cdot e^{(2x)} e^{(-2x)} + e^{-4x}}{4}\right)$$

$$\frac{1}{(D^3-4D)} = 2\left(\frac{e^{4x} + \cancel{2e^{2x}} \cdot \cancel{e^{-2x}} + e^{-4x}}{4}\right)$$

$$\frac{1}{(D^3-4D)} \cdot \left(\frac{e^{4x} + 2 + e^{-4x}}{2}\right)$$

denominator 0.

$$y = \frac{1}{2}\left[\frac{1}{(D^3-4D)} \cdot e^{4x} + \frac{2x}{(D^3-4D)} \cdot e^{0x} + \frac{1}{(D^3-4D)} \cdot e^{-4x}\right]$$

$$= \frac{1}{2}\left[\frac{1}{(D^3-4D)} \cdot e^{4x} + \frac{2x}{3D^2-4} \cdot e^{0x} + \frac{1}{(D^3-4D)} \cdot e^{-4x}\right]$$

$$= \frac{1}{2}\left[\frac{1}{4^3-4(4)} \cdot e^{4x} + \frac{2x}{0-4} \cdot e^{0x} + \frac{1}{(-4)^3+4(-4)} \cdot e^{-4x}\right]$$

$$= \frac{1}{2}\left[\frac{1}{48} \cdot e^{4x} + \frac{2x}{-4} - \frac{1}{48} \cdot e^{-4x}\right]$$

$$= \frac{1 \times 1}{2 \cdot 48}\left[\right.$$

$$\frac{1}{96}\left[(e^{4x} - e^{-4x}) - \frac{x}{2} = \frac{1}{48}\left(\frac{e^{4x} - e^{-4x}}{2}\right) - \frac{x}{2}\right.$$

$GS = CF + PI$.

$$y = \left[C_1 + C_2 e^{2x} + C_3 e^{-2x}\right] + \left[\frac{1}{48}\left(\frac{e^{4x} - e^{-4x}}{2}\right) - \frac{x}{2}\right]$$

$$= \left[c_1 + c_2 e^{2x} + c_3 e^{-2x}\right] + \left[\frac{1}{48}(\sinh 4x) - \frac{x}{2}\right].$$

Particular Integral. Types.

* when $X = e^{ax} = RHS$.

Type 2

Trig ÷ where sq comes sub the

$RHS = X = \sin ax / \cos ax$.

$$\frac{1}{f(D^2)} \cdot \sin ax = \cancel{\frac{1}{f(\sin ax)}} \; \frac{1}{f(-a^2)} \cdot \sin ax.$$

$$\frac{1}{f(D^2)} \cdot \cos ax = \frac{1}{f(-a^2)} \cdot \cos ax.$$

trig ÷ type 2.

(1) Solve: $(D^2 - 5D + 6)y = \sin 3x$.

Auxilliary eqn is

$$(D^2 - 5D + 6) = 0.$$

$$D = 2, 3.$$

CF is

$$\boxed{y = c_1 e^{2x} + c_2 e^{3x}}$$

for PI.

$$y = \frac{1}{(D^2-5D+6)} \cdot \sin 3x.$$

$$\therefore \quad y = \frac{1}{(-9-5D+6)} \cdot \sin 3x.$$

$$= \frac{1}{-5D-3} \cdot \sin 3x.$$

expecting D^2 co take Rationaly.

$$= \frac{1}{(-5D-3)} \times \frac{(-5D+3)}{(-5D+3)} \cdot \sin 3x.$$

$\boxed{(a^2-b^2)}$

$$= \frac{(-5D+3)}{(-5D)^2-(3)^2} \times \sin 3x$$

$$= \frac{(-5D+3)}{25D^2-9} \cdot (\sin 3x).$$

$(-9).$

$$= \frac{-5D+3}{25(-9)-9} \cdot (\sin 3x).$$

$D = \frac{d}{dx}.$

$$= \frac{(-5D+3)}{-225-9} (\sin 3x).$$

$$\frac{(-5D+3)}{(-234)} \cdot \sin 3x.$$

$$= -\frac{1}{234}\left[-5 \cdot \frac{d}{dx}(\sin 3x) + 3 \sin 3x\right]$$

$\frac{-1}{234}\left[-5\cos 3x \cdot 3 + 3\sin 3x\right].$

$= \frac{-1}{234}\left[-15\cos 3x + 3\sin 3x\right].$

$= \frac{15}{234} \cdot (\cos 3x) - \frac{3}{234}(\sin 3x) = y.$

$GS = CF + PI.$

② Solve: $(D^3+D)y = \cos x.$

AE: $D^3 + D = 0.$

$D(D^2+1) = 0.$

$D = 0, i, -i$

$\therefore CF = y_1 = c_1 e^{0x} + e^{0x}\left[c_2 \cos x + c_3 \sin x\right].$

$= \boxed{c_1 + c_2\cos x + c_3 \sin x} = CF.$

for P.I.

$y = \frac{1}{(D^3+D)} \cdot \cos x.$

$y = \frac{1}{D(D^2+1)} \cdot \cos x.$

$\frac{1}{(D^2+1)} \cdot \frac{1}{D} \cdot \cos x.$

$\frac{1}{(D^2+1)} \cdot \frac{1}{D} \cdot \frac{1}{D} \cos x$

$\frac{1}{(D^2+1)} \cdot \frac{D}{D^2} \cdot \cos(x).$

$\frac{1}{(D^2+1)} \cdot \frac{D}{(-1)} \cdot \cos x.$

$= \frac{1}{(D^2+1)} \cdot \frac{(-\sin x)}{-1}.$

$= \frac{1}{(D^2+1)} \cdot \sin(x).$

$-1+1$

$$= \frac{x}{2D} \cdot \sin x.$$

$$\boxed{\begin{aligned} D &\to \frac{d}{dx}. \\ \frac{1}{D} &= \int. \end{aligned}}$$

$$\frac{x}{2} \cdot \frac{1}{D} \sin x.$$

$$y = \frac{x}{2} \cdot \frac{1}{D} \sin x$$

$$\frac{x}{2} \int \sin x \, dx.$$

$$\frac{x}{2} [-\cos x].$$

$$\therefore y = -\frac{x \cos x}{2}.$$

$$GS = CF + PI.$$

$$\therefore y = [C_1 + C_2 \cos x + C_3 \sin x] + \left[-\frac{x \cos x}{2}\right].$$

$\star$

PI Type 3

$$RHS = X = x^m$$

$$PI = y = \frac{1}{f(D)} \cdot x^m$$

$$= \frac{1}{[1 + \phi(D)]} \cdot x^m$$

$$= [1 + \phi(D)]^{-1} \cdot x^m$$

$$=$$

$(f(x))$ Function of x.

being term 1.

$1 + \phi$

$$[1+x]^{-1} = 1 - x + x^2 - x^3 + x^4 \ldots$$

$$[1-x]^{-1} = 1 + x + x^2 + x^3 + x^4 \ldots$$

① Solve : $(D^3-3D+2)y = x$.

$AE = D^3-3D+2 = 0$.

$\therefore D = -2, 1, 1$.

$CF = \boxed{y = c_1 e^{-2x} + [c_2 + c_3 x]e^{x}}$.

for $\underline{PI}$,

$(D^3-3D+2)y = x$ → Algebraic (Type 3).

$y = \dfrac{1}{(D^3-3D+2)} \cdot x$.

[series wala type].

$\therefore y = \dfrac{1}{(D^3-3D+2)} \cdot x$.

make 1 in denomid so divide with const on up & down.

$\boxed{\dfrac{1}{\left[1+\left(\dfrac{D^3-3D}{2}\right)\right]}}$.

$= \dfrac{\frac{1}{2}}{\left[\frac{D^3}{2} - \frac{3D}{2} + \frac{2}{2}\right]} \cdot x$.

$\dfrac{1}{2} \cdot \dfrac{1}{\left[1+\left(\dfrac{D^3-3D}{2}\right)\right]} x$.

$\dfrac{1}{2}\left[1+\left(\dfrac{D^3-3D}{2}\right)\right]^{-1} x$.

$\therefore [1+x]^{-1} = 1 - x + x^2 - x^3 + x^4 \ldots$

$y = \frac{1}{2}\left[1 - \left(\frac{D^3 - 3D}{2}\right) + \left(\frac{D^3 - 3D}{2}\right)^2 \ldots\right] x$

$D(x) = \frac{d}{dx}(x) = 1$

$D^2(x) = \frac{d^2}{dx^2}(x) = 0$

$y = \frac{1}{2}\left[1 + \frac{3D}{2}\right] x$

$y = \frac{1}{2}\left[x + \frac{3}{2}(1)\right]$

$\therefore y = \frac{x}{2} + \frac{3}{4} = PI$

$GS = y = CF + PI$

$\boxed{c_1 e^{-2x} + [c_2 + c_3 x] e^{x} + \frac{x}{2} + \frac{3}{4}}$

$[1+x]^{-1}$

$\therefore [1+x]^{-1} =$

$\frac{d}{dx}(x) = 1$

Type 3 Rev.

$\frac{1}{f(D)} \cdot x^m = \frac{1}{[1+\phi(D)]} \cdot x^m$

$= [1+\phi(D)]^{-1} x^m$

$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 \ldots$

$(1-x)^{-1} \; 1 + x + x^2 + x^3 \ldots$

(2) $\frac{d^3y}{dx^3} - 2\frac{dy}{dx} + 4y = (3x^2 - 5x + 2)$.

$(D^3y - 2Dy + 4y) = 0$.

$(D^3 - 2D + 4)y = 0$.

$y = -2, (1+i), (1-i)$.

$CF = c_1 e^{-2x} + \cancel{e^x\{(c_2 + c_3 x)} + e^x[c_2 \cos x + c_3 \sin x]$

$PI = y = \frac{1}{(D^3 - 2D + 4)} \cdot (3x^2 - 5x + 4)$.

$y = \frac{\frac{1}{4}}{\left(\frac{D^3}{4} - \frac{2D}{4} + \frac{4}{4}\right)} = \frac{\frac{1}{4}}{\left(\frac{D^3}{4} - \frac{2D}{4} + 1\right)} \cdot (3x^2 - 5x + 4)$.

$= \frac{1}{4}\left[1 + \left(\frac{D^3 - 2D}{4}\right)\right]^{-1} \cdot (3x^2 - 5x + 4)$.

$(1 + x)^{-1}$ series

$\frac{1}{4}\left[1 - \left(\frac{D^3 - 2D}{4}\right)^{1} + \left(\frac{D^3 - 2D}{4}\right)^{2} - \left(\frac{D^3 - 2D}{4}\right)^{3} + \cdots\right] (3x^2 - 5x$

$$\boxed{\begin{aligned} &D(3x^2 - 5x + 4) = 6x - 5. \\ &D^2(3x^2 - 5x + 4) = 6. \end{aligned}}$$

$$= \frac{1}{4}\left[1 + \frac{2D}{4} + \frac{4D^2}{16}\right](3x^2 - 5x + 2).$$

$$= \frac{1}{4}\left\{(3x^2 - 5x + 2) + \frac{1}{2}(6x - 5) + \frac{1}{4}(6)\right]$$

$$\frac{1}{4}\left\{3x^2 - 5x + 2 + 3x - \frac{5}{2} + \frac{3}{2}\right].$$

$$\frac{1}{4}\left\{3x^2 - 5x + 2 + 3x - 1\right].$$

$$= \frac{1}{4}\left[3x^2 - 2x + 1\right].$$

$$\therefore GS = CF + PI.$$

$$= C_1 e^{-2x} + e^{x}(C_2 \cos x + C_3 \sin x) + \frac{1}{4}(3x^2 - 2x + 1)$$

Type 4.

$$\frac{1}{f(D)} \cdot e^{ax} \cdot V = e^{ax} \cdot \frac{1}{f(D+a)} \cdot V.$$

(OR).

$$\frac{1}{f(D)} \cdot e^{-ax} \cdot V = e^{-ax} \cdot \frac{1}{f(D-a)} \cdot V.$$

① Solve : $(D^2-3D+2)y = \underset{\text{Alg}}{\underline{x^2}} \cdot \underset{\text{exp}}{\underline{e^{2x}}}$ ... type 4.

$AE = (D^2-3D+2)y = 0.$

$D = 1,2.$

$$\boxed{CF = c_1 e^{x} + c_2 e^{2x}.}$$

$PI = \dfrac{1}{D^2-3D+2} \cdot (x^2 \cdot e^{2x}).$

put $D = D+a$.

$e^{2x} \cdot \dfrac{1}{[(D+2)^2 - 3(D+2)+2]} \cdot x^{2}.$

$y = e^{2x} \dfrac{1}{D^2+4D+4-3D-6+2} \cdot x^2.$

$= e^{2x} \cdot \dfrac{1}{D^2+D} \cdot x^2.$

$= e^{2x} \cdot \dfrac{1}{D\{D+1\}} \cdot x^2.$

$e^{2x} \cdot \dfrac{1}{(D+1)} \cdot \dfrac{1}{D} \cdot x^2.$

$e^{2x} \dfrac{1}{(D+1)} \cdot \int x^2 dx.$

$$e^{2x} \cdot \frac{1}{(D+1)} \cdot \left(\frac{x^3}{3}\right)$$

$$= \frac{e^{2x}}{3} \cdot [1+D]^{-1} \cdot x^3$$

$$= \frac{e^{2x}}{3}\left[1 - D + D^2 - D^3 + \dots\right] x^3$$

$$= \frac{e^{2x}}{3}\left[x^3 - (3x^2) + 6x\right.$$

$$= \frac{e^{2x}}{3}\left[x^3 - 3x^2 + 6x - 6\right].$$

$GS = CF + PI$.

$$= \boxed{c_1 e^{x} + c_2 e^{2x} + \frac{e^{2x}}{3}(x^3 - 3x^2 + 6x - 6)} \;//.$$

(2) Solve $\frac{d^2y}{dx^2} + 2y = x^2 \cdot e^{3x} + e^{x} - \cos 2x$.

$AE = D^2y + 2y = 0$.

$D = \pm\sqrt{2}i$.

$CF = y = e^{0x}\left[\cos\sqrt{2}x + \sin\sqrt{2}x\right].$

$$\boxed{y = \cos\sqrt{2}x + \sin\sqrt{2}x}$$

for PI - I

$$y_1 = \frac{1}{(D^2+2)} \cdot (x^2 \cdot e^{3x})$$

$$e^{3x} \cdot \frac{1}{D^2+2} \cdot x^2$$

$$e^{3x} \cdot \frac{1}{[(D+3)^2+2]} \cdot x^2$$

$$e^{3x} = \frac{1}{D^2+6D+9+2} \cdot x^2$$

$$= e^{3x} \cdot \frac{1}{D^2+6D+11} \cdot x^2$$

$$= e^{3x} \cdot \frac{\frac{1}{11}}{\left(\frac{D^2+6D}{11}\right)+1} \cdot x^2$$

$$= e^{3x} \cdot \left[1+\left(\frac{D^2+6D}{11}\right)\right]^{-1} \cdot x^2$$

$$= \frac{e^{3x}}{11}\left[1-\left(\frac{D^2+6D}{11}\right)+\left(\frac{D^2+6D}{11}\right)^2 - \dots\right]x^2$$

$$= \frac{e^{3x}}{11}\left[1-\frac{D^2}{11}-\frac{6D}{11}+\frac{36D^2}{121}\right] \cdot x^2$$

$$\frac{e^{3x}}{11}\left[x^2-\frac{2}{11}-\frac{12x}{11}+\frac{72}{121}\right]$$

$$= \frac{e^{3x}}{11}\left[x^2-\frac{12x}{11}+\frac{50}{121}\right]$$

$\therefore y_2 = \frac{1}{(D^2+2)} \cdot e^{x}$

$= \frac{1}{(1+2)} \cdot e^{x}$

$= \frac{1}{3} \cdot e^{x}$

$y_3 = \frac{1}{(D^2+2)} \cdot \cos 2x$

$= \frac{1}{-4+2} \cdot \cos 2x$

$= \frac{1}{-2} \cos 2x$

$\therefore PI = y_1 + y_2 + y_3$

$= \frac{e^{3x}}{11}\left[x^2 - \frac{12x}{11} + 50\right]$

$+ \frac{e^{x}}{3} - \left(-\frac{\cos 2x}{2}\right)$

$= \frac{e^{3x}}{11}\left[x^2 - \frac{12x}{11} + \frac{50}{121}\right]$

$+ \frac{e^{x}}{3} + \frac{\cos 2x}{2}$

$GS = y = CF + PI$

$\boxed{\text{Type 5}}$ $\boxed{\text{Alg + Trig}}$

$\frac{1}{f(D)} \cdot XV$

$\frac{1}{f(D)} \cdot XV$

$= \left\{X - \frac{1}{f(D)} \cdot f'(D)\right\} \frac{1}{f(D)} V$

(1) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^{x}\sin x$

$AE = D^2 - 2D + 1 = 0$

$\therefore (D-1)^2 = 0$

$D = 1, 1$

CF is $\boxed{y = [C_1 + C_2 x]e^{x}}$

$\therefore PI = y = \frac{1}{(D^2-2D+1)} \cdot xe^{x}\sin x$

$$y = e^x \cdot \frac{1}{(D+1)^2 - 2(D+1) + 1} \cdot x \sin x$$

$$= e^x \cdot \frac{1}{D^2 + \cancel{2D} + 1 - \cancel{2D} - 2 + 1} \cdot x \sin x$$

$$e^x \cdot \frac{1}{D^2} \cdot x \sin x$$

$$= e^x \cdot \int x \cdot \sin x$$

Type 5 1tent

$$e^x \left[x - \frac{1}{D^2} \cdot 2D \right] \frac{1}{D^2} \cdot \sin x$$

$$= e^x \left[x - \frac{1}{D^2} \cdot 2D \right] \frac{1}{D} \cdot (-\cos x)$$

$$= e^x \left[x - \frac{1}{D^2} \cdot 2D \right] (-\sin x)$$

$$= -e^x \left[x - \frac{1}{D^2} \cdot 2D \right] (+\sin x)$$

$$= -e^x \left[x \sin x - \frac{1}{D^2} \cdot 2(+\cos x) \right]$$

$$= -e^x \left[x \sin x \mp 2 \cdot \frac{1}{D^2} \cdot \cos x \right]$$

$$= e^x \left[x \sin x \mp 2 \cdot \frac{1}{D} (\sin x) \right]$$

$$= -e^x \left[x \sin x \mp 2(-\cos x) \right]$$

$$= -e^x \left[x \sin x + 2 \cos x \right] = PI$$

$\therefore$ GS:

$y = CF + PI$

$\therefore y = (C_1 + C_2 x) e^x$

$- \cancel{+}\, e^x [2 \cos x - x \ldots]$