DATA REPRESENTATION

Data Types

Complements

Fixed Point Representations

Floating Point Representations

Other Binary Codes

Error Detection Codes

DATA REPRESENTATION

Information that a Computer is dealing with

*** Data**
- **Numeric Data**
  Numbers(Integer, real)
- **Non-numeric Data**
  Letters, Symbols

*** Relationship between data elements**
- **Data Structures**
  Linear Lists, Trees, Rings, etc

*** Program(Instruction)**

NUMERIC DATA REPRESENTATION

Data

Numeric data - numbers(integer, real)
Non-numeric data - symbols, letters

Number System

Nonpositional number system
- **Roman number system**

Positional number system
- **Each digit position has a value called a *weight* associated with it**
- **Decimal, Octal, Hexadecimal, Binary**

Base (or radix) R number
- **Uses R distinct symbols for each digit**
- **Example** $A_R = a_{n-1}\, a_{n-2} \ldots a_1\, a_0\, .a_{-1} \ldots a_{-m}$
- $V(A_R) = \sum_{i=-m}^{n-1} a_i R^i$

Radix point(.) separates the integer portion and the fractional portion

R = 10 Decimal number system, R = 2 Binary
R = 8 Octal, R = 16 Hexadecimal

WHY POSITIONAL NUMBER SYSTEM IN THE DIGITAL COMPUTERS ?

Major Consideration is the *COST* and *TIME*

- **Cost of building *hardware***
  Arithmetic and Logic Unit, CPU,Communications
- **Time to processing**

Arithmetic - Addition of Numbers - *Table for Addition*

*** Non-positional Number System**
- **Table for addition is infinite**
  --> Impossible to build, very expensive even if it can be built

*** Positional Number System**
- **Table for Addition is finite**
  --> Physically realizable, but cost wise the smaller the table size, the less expensive --> Binary is favorable to Decimal

Binary Addition Table

	0	1
0	0	1
1	1	10

Decimal Addition Table

	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

REPRESENTATION OF NUMBERS

POSITIONAL NUMBERS

Decimal	Binary	Octal	Hexadecimal
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Binary, octal, and hexadecimal conversion

1	**2**	**7**	**5**	**4**	**3**		**Octal**
1	**0 1 0**	**1 1 1**	**1 0 1**	**1 0 0**	**0 1 1**		**Binary**

1 0 1 0	**1 1 1 1**	**0 1 1 0**	**0 0 1 1**	**Binary**
A	**F**	**6**	**3**	**Hexa**

CONVERSION OF BASES

Base R to Decimal Conversion

$$A = a_{n-1}\ a_{n-2}\ \ a_{n-3} \ldots a_0 \,.\, a_{-1} \ldots a_{-m}$$

$$V(A) = \Sigma\, a_k R^k$$

$(736.4)_8 = 7 \times 8^2 + 3 \times 8^1 + 6 \times 8^0 + 4 \times 8^{-1}$

$= 7 \times 64 + 3 \times 8 + 6 \times 1 + 4/8 = (478.5)_{10}$

$(110110)_2 = \ldots = (54)_{10}$

$(110.111)_2 = \ldots = (6.785)_{10}$

$(F3)_{16} = \ldots = (243)_{10}$

$(0.325)_6 = \ldots = (0.578703703 \ldots\ldots\ldots\ldots\ldots)_{10}$

Decimal to Base R number

- Separate the number into its *integer* and *fraction* parts and convert each part separately.
- Convert *integer part* into the base R number

  --> successive divisions by R and accumulation of the remainders.
- Convert *fraction part* into the base R number

  --> successive multiplications by R and accumulation of integer digits

EXAMPLE

Convert 41.6875_{10} to base 2.

Integer = 41

41	
20	1
10	0
5	0
2	1
1	0
0	1

$(41)_{10} = (101001)_2$

Fraction = 0.6875

$$\begin{array}{r} 0.6875 \\ \underline{\times \quad 2} \\ 1.3750 \\ \underline{\times \quad 2} \\ 0.7500 \\ \underline{\times \quad 2} \\ 1.5000 \\ \underline{\times \quad 2} \\ 1.0000 \end{array}$$

$(0.6875)_{10} = (0.1011)_2$

$(41.6875)_{10} = (101001.1011)_2$

<u>Exercise</u>

Convert $(63)_{10}$ to base 5: $(223)_5$
Convert $(1863)_{10}$ to base 8: $(3507)_8$
Convert $(0.63671875)_{10}$ to hexadecimal: $(0.A3)_{16}$

COMPLEMENT OF NUMBERS

Complements

- to convert positive to negative or vice versa

Two types of complements for base R number system:

- R's complement and (R-1)'s complement

The (R-1)'s Complement

Subtract each digit of a number from (R-1)

Example

- 9's complement of 835_{10} is 164_{10}
- 1's complement of 1010_2 is 0101_2 (bit by bit complement operation)

The R's Complement

Add 1 to the low-order digit of its (R-1)'s complement

Example

- 10's complement of 835_{10} is $164_{10} + 1 = 165_{10}$
- 2's complement of 1010_2 is $0101_2 + 1 = 0110_2$

FIXED POINT NUMBERS

Numbers: Fixed Point Numbers and Floating Point Numbers

Binary Fixed-Point Representation

$X = x_n x_{n-1} x_{n-2} \ldots x_1 x_0 . x_{-1} x_{-2} \ldots x_{-m}$

Sign Bit(x_n): 0 for positive - 1 for negative

Remaining Bits($x_{n-1} x_{n-2} \ldots x_1 x_0 . x_{-1} x_{-2} \ldots x_{-m}$)

- Following 3 representations

Signed magnitude representation
Signed 1's complement representation
Signed 2's complement representation

Example: Represent +9 and -9 in 7 bit-binary number

Only one way to represent +9 ==> 0 001001

Three different ways to represent -9:

In signed-magnitude: 1 001001

In signed-1's complement: 1 110110

In signed-2's complement: 1 110111

In general, in computers, fixed point numbers are represented either integer part only or fractional part only.

CHARACTERISTICS OF 3 DIFFERENT REPRESENTATIONS

Complement

Signed magnitude: Complement *only* the sign bit
Signed 1's complement: Complement *all* the bits including sign bit
Signed 2's complement: Take the 2's complement of the number, *including* its sign bit.

Maximum and Minimum Representable Numbers and Representation of Zero

$$X = x_n\ x_{n-1} \ldots x_0 \,.\, x_{-1} \ldots x_{-m}$$

Signed Magnitude

Max:	$2^n - 2^{-m}$	011 ... 11.11 ... 1
Min:	$-(2^n - 2^{-m})$	111 ... 11.11 ... 1
Zero:	+0	000 ... 00.00 ... 0
	-0	100 ... 00.00 ... 0

Signed 1's Complement

Max:	$2^n - 2^{-m}$	011 ... 11.11 ... 1
Min:	$-(2^n - 2^{-m})$	100 ... 00.00 ... 0
Zero:	+0	000 ... 00.00 ... 0
	-0	111 ... 11.11 ... 1

Signed 2's Complement

Max:	$2^n - 2^{-m}$	011 ... 11.11 ... 1
Min:	-2^n	100 ... 00.00 ... 0
Zero:	0	000 ... 00.00 ... 0

ARITHMETIC ADDITION: SIGNED MAGNITUDE

[1] Compare their signs
[2] If two signs are the *same* , *ADD* the two magnitudes - Look out for an *overflow*
[3] If *not the same* , compare the relative magnitudes of the numbers and then *SUBTRACT* the smaller from the larger --> need a subtractor to add
[4] Determine the sign of the result

6 + 9

```
   6    0110
+) 9    1001
-------------------
  15    1111 -> 01111
```

-6 + 9

```
   9    1001
-) 6    0110
-------------------
   3    0011 -> 00011
```

6 + (- 9)

```
   9   1001
-) 6   0110
-------------------
 - 3   0011 -> 10011
```

-6 + (-9)

```
   6   0110
+) 9   1001
-------------------
 -15   1111 -> 11111
```

Overflow 9 + 9 or (-9) + (-9)

```
          9     1001
      +)  9     1001
-------------------------
overflow  (1)0010
```

ARITHMETIC ADDITION: SIGNED 2's COMPLEMENT

Add the two numbers, including their sign bit, and discard any carry out of leftmost (sign) bit

Example

	6	0	0110
+)	9	0	1001
	15	0	1111

	-6	1	1010
+)	9	0	1001
	3	0	0011

	6	0	0110
+)	-9	1	0111
	-3	1	1101

	-9	1	0111
+)	-9	1	0111
	-18	(1)0	1110

$x'_{n-1}y'_{n-1}s_{n-1}$
$(c_{n-1} \oplus c_n)$

overflow

	9	0	1001
+)	9	0	1001
	18	1	0010

$x_{n-1}y_n s'_{n-1}$
$(c_{n-1} \oplus c_n)$

2 operands have the same sign and the result sign changes
$x_{n-1}y_{n-1}s'_{n-1} + x'_{n-1}y'_{n-1}s_{n-1}$

ARITHMETIC ADDITION: SIGNED 1's COMPLEMENT

Add the two numbers, including their sign bits.

- If there is a carry out of the most significant (sign) bit, the result is incremented by 1 and the carry is discarded.

Example

```
      6   0  0110
+)   -9   1  0110
-----------------
     -3   1  1100
```

end-around carry

```
     -6   1   1001
+)    9   0   1001
------------------
   (1) 0(1)0010
+)               1
------------------
      3   0   0011
```

not overflow $(c_{n-1} \oplus c_n) = 0$

```
     -9   1  0110
+)   -9   1  0110
-----------------
        (1)0  1100
+)               1
-----------------
          0  1101
```

```
      9   0    1001
+)    9   0    1001
-------------------
          1  (1)0010
```

overflow
$(c_{n-1} \oplus c_n)$

COMPARISON OF REPRESENTATIONS

* **Easiness of negative conversion**

  S + M > 1's Complement > 2's Complement

* **Hardware**

  - S+M: Needs an adder and a subtractor for Addition
  - 1's and 2's Complement: Need only an adder

* **Speed of Arithmetic**

  2's Complement > 1's Complement(end-around C)

* **Recognition of Zero**

  2's Complement is fast

ARITHMETIC SUBTRACTION

Subtraction

Add complement of the subtrahend to the minuend including the sign bits.

Take the complement of the subtrahend

$$(\pm A) - (-B) = (\pm A) + B$$

$$(\pm A) - B = (\pm A) + (-B)$$

FLOATING POINT NUMBER REPRESENTATION

* The location of the fractional point is not fixed to a certain location
* The range of the representable numbers is wide

$$F = EM$$

m_n	$e_k e_{k-1} \ldots e_0$	$m_{n-1} m_{n-2} \ \ldots \ m_0 \,.\, m_{-1} \ \ldots \ m_{-m}$
sign	exponent	mantissa

- Mantissa
  Signed fixed point number, either an integer or a fractional number

- Exponent
  Designates the position of the radix point

Decimal Value

$$V(F) = V(M) * R^{V(E)}$$

M: Mantissa
E: Exponent
R: Radix

FLOATING POINT NUMBERS

Example

sign		sign	
0	.1234567	0	04
	mantissa		exponent

==> $+.1234567 \times 10^{+04}$

Note:

In Floating Point Number representation, only Mantissa(M) and Exponent(E) are explicitly represented. The Radix(R) and the position of the Radix Point are implied.

Example

A binary number +1001.11 in 16-bit floating point number representation (6-bit exponent and 10-bit fractional mantissa)

0	0 00100	100111000
Sign	Exponent	Mantissa

or

0	0 00101	010011100

CHARACTERISTICS OF FLOATING POINT NUMBER REPRESENTATIONS

Normal Form

- **There are many different floating point number representations of the same number**
  --> Need for a unified representation in a given computer

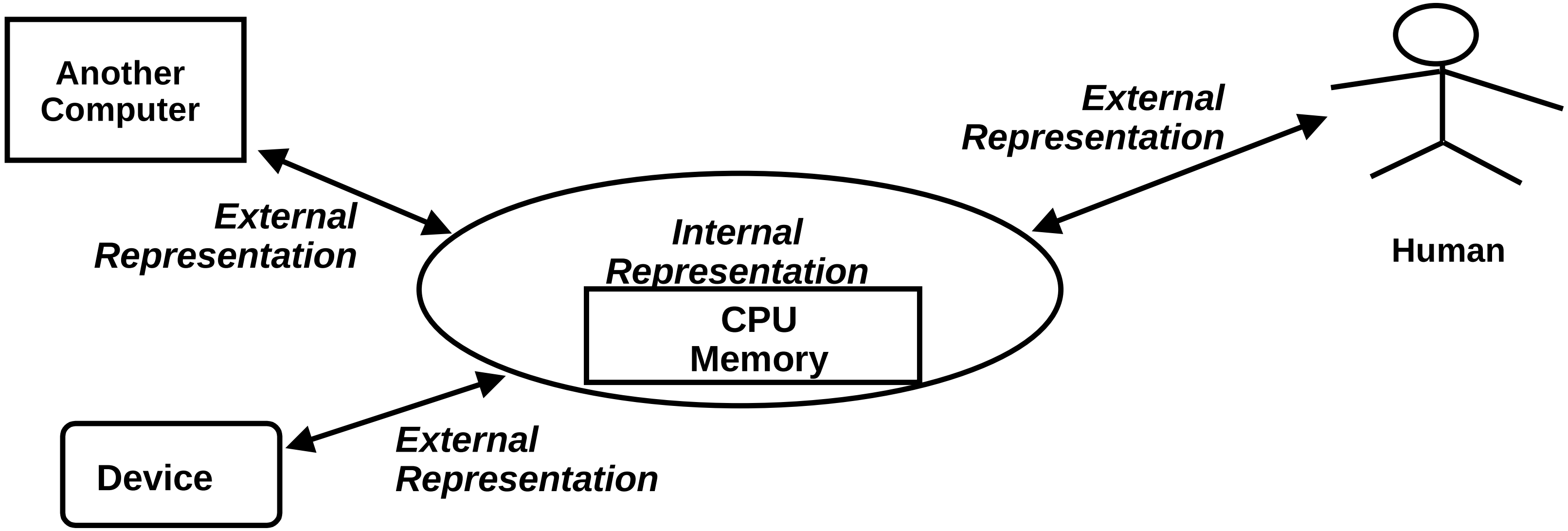
- ***the most significant position of the mantissa contains a non-zero digit***

Representation of Zero

- **Zero**
  Mantissa = 0

- **Real Zero**
  Mantissa = 0
  Exponent
  = smallest representable number
  which is represented as
  00 ... 0
  <-- Easily identified by the hardware

INTERNAL REPRESENTATION AND EXTERNAL REPRESENTATION



External Representations	Internal Representations
- Presentability - Efficiency Communication Reliability - Easy to handle - BCD, ASCII, EBCDIC	- Efficiency Memory space Processing time - Easy to convert to external representation - Fixed and Floating points

EXTERNAL REPRESENTATION

Numbers

Most of numbers stored in the computer are eventually changed by some kinds of calculations

--> *Internal Representation* for calculation efficiency

--> Final results need to be converted to as *External Representation* for presentability

Alphabets, Symbols, and some Numbers

Elements of these information do not change in the course of processing

--> No needs for Internal Representation since they are not used for calculations

--> External Representation for processing and presentability

Example

Decimal Number: 4-bit Binary Code
BCD(Binary Coded Decimal)

Decimal	BCD Code
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

OTHER DECIMAL CODES

Decimal	BCD(8421)	2421	84-2-1	Excess-3
0	0000	0000	0000	0011
1	0001	0001	0111	0100
2	0010	0010	0110	0101
3	0011	0011	0101	0110
4	0100	0100	0100	0111
5	0101	1011	1011	1000
6	0110	1100	1010	1001
7	0111	1101	1001	1010
8	1000	1110	1000	1011
9	1001	1111	1111	1100

Note: 8,4,2,-2,1,-1 in this table is the weight associated with each bit position.

d3 d2 d1 d0: symbol in the codes

BCD: d3 x 8 + d2 x 4 + d1 x 2 + d0 x 1
==> 8421 code.
2421: d3 x 2 + d2 x 4 + d1 x 2 + d0 x 1
84-2-1: d3 x 8 + d2 x 4 + d1 x (-2) + d0 x (-1)
Excess-3: BCD + 3

BCD: It is difficult to obtain the 9's complement.
However, it is easily obtained with the other codes listed above.

GRAY CODE

* Characterized by having their representations of the binary integers differ in only one digit between consecutive integers

* Useful in analog-digital conversion.

4-bit Gray codes

Decimal number	Gray g_3	g_2	g_1	g_0	Binary b_3	b_2	b_1	b_0
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
2	0	0	1	1	0	0	1	0
3	0	0	1	0	0	0	1	1
4	0	1	1	0	0	1	0	0
5	0	1	1	1	0	1	0	1
6	0	1	0	1	0	1	1	0
7	0	1	0	0	0	1	1	1
8	1	1	0	0	1	0	0	0
9	1	1	0	1	1	0	0	1
10	1	1	1	1	1	0	1	0
11	1	1	1	0	1	0	1	1
12	1	0	1	0	1	1	0	0
13	1	0	1	1	1	1	0	1
14	1	0	0	1	1	1	1	0
15	1	0	0	0	1	1	1	1

GRAY CODE - ANALYSIS

Letting $g_n g_{n-1} \ldots g_1\ g_0$ be the (n+1)-bit Gray code for the binary number $b_n b_{n-1} \ldots b_1 b_0$

$$g_i = b_i \oplus b_{i+1} \quad , \quad 0 \le i \le n-1$$
$$g_n = b_n$$

and

$$b_{n-i} = g_n \oplus g_{n-1} \oplus \ldots \oplus g_{n-i}$$
$$b_n = g_n$$

Reflection of Gray codes

ε	0	0 0	0 00	0 000
	1	0 1	0 01	0 001
		1 1	0 11	0 011
		1 0	0 10	0 010
			1 10	0 110
			1 11	0 111
			1 01	0 101
			1 00	0 100
				1 100
				1 101
				1 111
				1 010
				1 011
				1 001
				1 101
				1 000

Note:

The Gray code has a reflection property
- **easy to construct a table without calculation,**
- **for any n: reflect case n-1 about a mirror at its bottom and prefix 0 and 1 to top and bottom halves, respectively**

CHARACTER REPRESENTATION ASCII

ASCII (American Standard Code for Information Interchange) Code

MSB (3 bits)

LSB (4 bits)

	0	1	2	3	4	5	6	7
0	NUL	DLE	SP	0	@	P	‘	P
1	SOH	DC1	!	1	A	Q	a	q
2	STX	DC2	“	2	B	R	b	r
3	ETX	DC3	#	3	C	S	c	s
4	EOT	DC4	$	4	D	T	d	t
5	ENQ	NAK	%	5	E	U	e	u
6	ACK	SYN	&	6	F	V	f	v
7	BEL	ETB	‘	7	G	W	g	w
8	BS	CAN	(	8	H	X	h	x
9	HT	EM	)	9	I	Y	I	y
A	LF	SUB	*	:	J	Z	j	z
B	VT	ESC	+	;	K	[	k	{
C	FF	FS	,	<	L	\	l	|
D	CR	GS	-	=	M	]	m	}
E	SO	RS	.	>	N	m	n	~
F	SI	US	/	?	O	n	o	DEL

CONTROL CHARACTER REPRESENTAION (ACSII)

NUL	Null	DC1	Device Control 1
SOH	Start of Heading (CC)	DC2	Device Control 2
STX	Start of Text (CC)	DC3	Device Control 3
ETX	End of Text (CC)	DC4	Device Control 4
EOT	End of Transmission (CC)	NAK	Negative Acknowledge (CC)
ENQ	Enquiry (CC)	SYN	Synchronous Idle (CC)
ACK	Acknowledge (CC)	ETB	End of Transmission Block (CC)
BEL	Bell	CAN	Cancel
BS	Backspace (FE)	EM	End of Medium
HT	Horizontal Tab. (FE)	SUB	Substitute
LF	Line Feed (FE)	ESC	Escape
VT	Vertical Tab. (FE)	FS	File Separator (IS)
FF	Form Feed (FE)	GS	Group Separator (IS)
CR	Carriage Return (FE)	RS	Record Separator (IS)
SO	Shift Out	US	Unit Separator (IS)
SI	Shift In	DEL	Delete
DLE	Data Link Escape (CC)		

(CC) Communication Control
(FE) Format Effector
(IS) Information Separator

ERROR DETECTING CODES

Parity System

- Simplest method for error detection
- One *parity* bit attached to the information
- *Even Parity* and *Odd Parity*

Even Parity

- One bit is attached to the information so that the total number of 1 bits is an even number

1011001 $\boxed{0}$
1010010 $\boxed{1}$

Odd Parity

- One bit is attached to the information so that the total number of 1 bits is an odd number

1011001 $\boxed{1}$
1010010 $\boxed{0}$

PARITY BIT GENERATION

Parity Bit Generation

For $b_6b_5 ... b_0$(7-bit information); even parity bit b_{even}

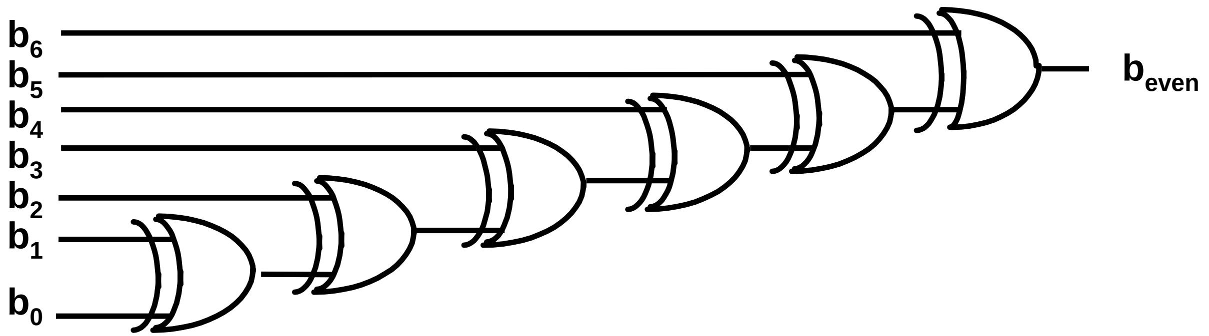
$$b_{even} = b_6 \oplus b_5 \oplus ... \oplus b_0$$

For even parity bit

$$b_{odd} = b_{even} \oplus 1 = b_{even}$$

PARITY GENERATOR AND PARITY CHECKER

Parity Generator Circuit(even parity)



Parity Checker