

15/11/21

UNIT-3

①

Noam chomsky gave a mathematical model of Grammar which is effective for writing Computer Language.

The four type of Grammars according to Noam chomsky are :

Grammar Type	Grammar Accepted	Language Accepted	Automaton
Type 0	unrestricted grammar	Recursively Enumerable Language.	<del>Enumerable</del> Turing machine
Type 1	Context sensitive Grammar	Context sensitive Language.	Linear Bounded Automaton
Type 2	Context free Grammar	Context free Language	Push down Automata
Type 3	Regular Grammar	Regular Language	finite state Automaton

Grammar :-

A Grammar 'G' can be formally described using 4 tuples as  $G = (V, T, S, P)$  where,

$V =$  Set of Variables or non-Terminal symbols

$T =$  Set of ~~Terminal~~ Terminal symbols

$S =$  Start symbol

$P =$  Production rules for Terminals and non-Terminals

A production rules has the form  $\alpha \rightarrow \beta$  (union) where  $\alpha$  and  $\beta$  are strings on  $V \cup T$  and atleast one symbol of ' $\alpha$ ' belongs to ' $V$ '

Example :- ~~G~~  $G = (S, A, B), (a, b), S, (S \rightarrow AB, A \rightarrow a, B \rightarrow b)$

$V = \{S, A, B\}$  non terminals

$P =$  Production rules

$T = \{a, b\}$  terminals

$S = S$  start symbol

$S \rightarrow AB$   
 $A \rightarrow a$   
 $B \rightarrow b$  } This is the production rules

Eg:-

$$S \rightarrow AB$$

$\rightarrow A$  (instead of A we can put a)

$\rightarrow aB$  (instead of B we can put b)

$$\rightarrow \underline{ab}$$

Now need to discuss Regular Grammar.

Regular Grammar:-

Regular Grammar can be divided into two types.

Right Linear Grammar :-

A Grammar is said to be Right Linear if all productions are of the form

$$A \rightarrow XB$$

$$A \rightarrow X$$

where  $A, B \in V$  and  $X \in T$  (Terminals)

(Terminals)



## Left Linear Grammar $\text{S} \rightarrow$

(9)

A grammar is said to be left linear if all productions are of the form

$$A \rightarrow BX$$

$$A \rightarrow x$$

where  $A, B \in V$  and  $x \in T$ .

Ex:

$$S \rightarrow abs|b$$

$S$  which is a non terminal symbol and right of  $S$  i.e,  $a$  and  $b$  are the terminal symbols so. above is the right linear grammar.

$$S \rightarrow sb|b$$

$S$  which is a non terminal symbol and  $b$  is a non terminal symbol. so this is a left linear grammar.



# 4/12/21 Unit-3 Ambiguity in CFG (1)

A Grammar  $G$  is ambiguous if there exists some string  $w \in L(G)$  for which there are two or more distinct derivation trees, or there are two or more distinct leftmost derivations.

Example:— Consider CFG  $S \rightarrow S+S | S * S | a | b$  and string  $w = a * a + b$ , and derivation as follows.

Solution—

first leftmost derivations for  $w = a * a + b$

$$S \Rightarrow S * S \quad (\text{using } S \rightarrow S * S)$$

$$\Rightarrow a * S \quad (\text{using } S \rightarrow a)$$

$$\Rightarrow a * S + S \quad (\text{using } S \rightarrow S + S)$$

$$\Rightarrow a * a + S \quad (\text{using } S \rightarrow a)$$

$$\Rightarrow a * a + b \quad (\text{using } S \rightarrow b)$$

Second leftmost derivation for  $w = a * a + b$

$S \Rightarrow S * S$  (using  $S \rightarrow S * S$ )

(2)

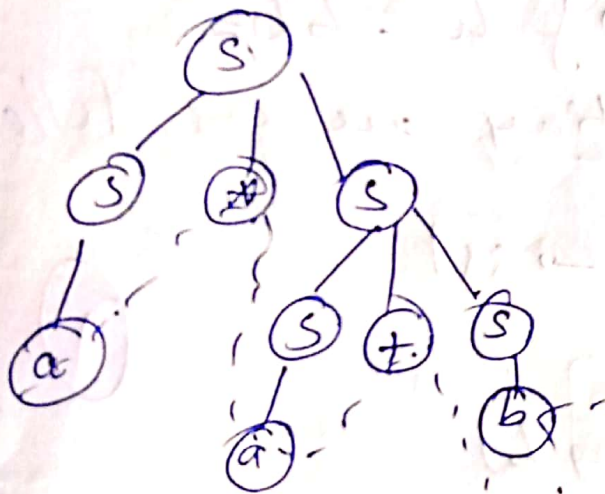
$\Rightarrow S * S + S$  (using  $S \rightarrow S + S$ )

$\Rightarrow a * S + S$  (using  $S \rightarrow a$ )

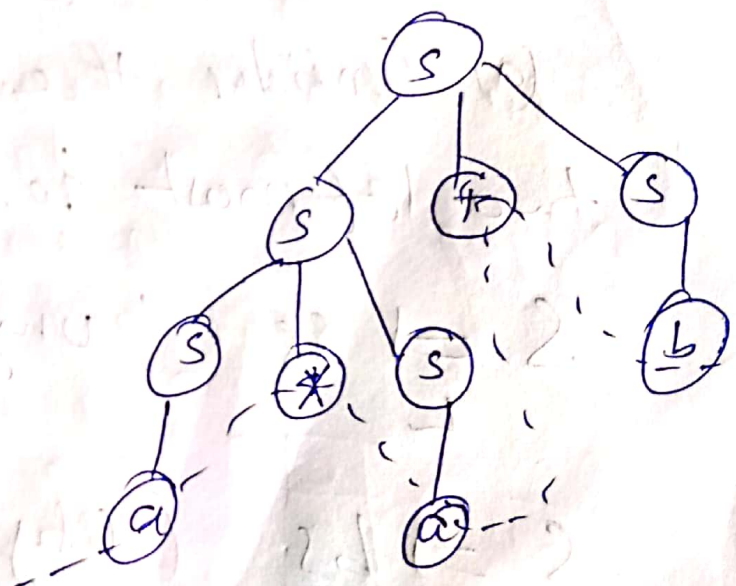
$\Rightarrow a * a + S$  (using  $S \rightarrow a$ )

$\Rightarrow a * a + b$  (using  $S \rightarrow b$ )

two distinct parse trees are shown figure (a) and figure (b)



(a)



(b)

Fig (a) Parse tree for  $a * a + b$

Fig (b) : parse tree for  $a * a + b$

Since there are two distinct leftmost derivations (two parse tree) for string  $w$ . Hence  $w$  is ambiguous there is ambiguity in Grammar  $G$ .



Ex:- Show that the following grammars are ambiguous, (3)

(a)  $S \rightarrow ss \mid a \mid b$

(b)  $S \rightarrow A \mid B \mid b$ ,

$A \rightarrow aAB \mid ab$ ,

$B \rightarrow abB \mid \epsilon$

Solution:-

(a) Consider the string  $w = bbb$ .  
two leftmost derivations are as follows

$S \Rightarrow ss$  (using  $S \rightarrow ss$ )

$S \Rightarrow bs$  (using  $S \rightarrow b$ )

$\Rightarrow bss$  (using  $S \rightarrow ss$ )

$\Rightarrow bbs$  (using  $S \rightarrow b$ )

$\Rightarrow bbb$  (using  $S \rightarrow b$ )



~~on other way~~

$S \Rightarrow SS$  (only  $S \rightarrow SS$ )

(4)

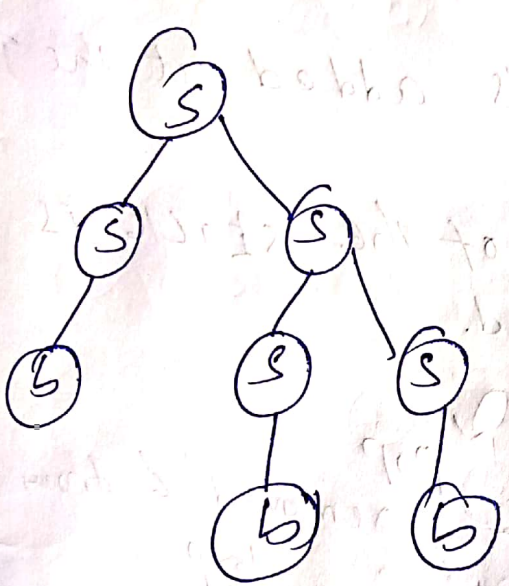
$\Rightarrow SSS$  (only  $S \rightarrow SS$ )

$\Rightarrow bSS$  (only  $S \rightarrow b$ )

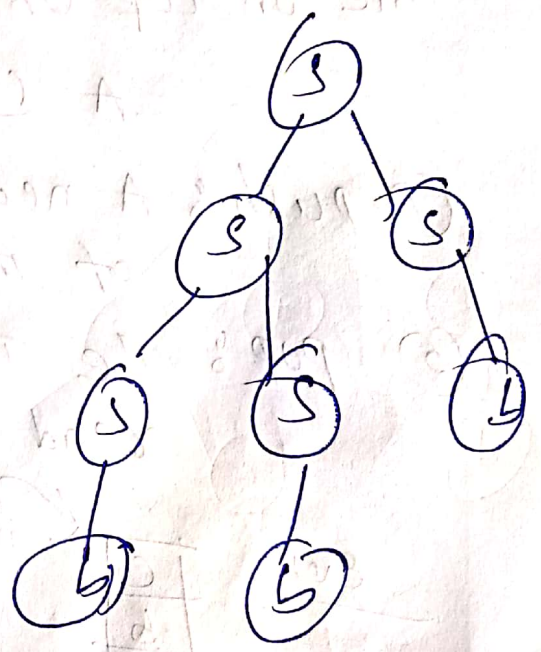
(2)

$\Rightarrow bbs$  (only  $S \rightarrow b$ )

$\Rightarrow bbb$  (only  $S \rightarrow b$ )



(a) Parse tree for bbb



Parse tree for bbb

$\therefore$ , when two grammars are ambiguous

# ② Context Free Language

①

## Derivation Tree

or

## Parse Tree

A Derivation Tree or Parse Tree is an ordered rooted tree that graphically represents the semantic information of strings derived from a Context free Grammar.

Example - For the Grammar  $G = \{V, \Sigma, P, S\}$   
where  $S \rightarrow OB$ ,  $A \rightarrow \epsilon | AA$ ,  
 $B \rightarrow \epsilon | AA$

Root vertex: must be labelled by the start symbol

Vertex: Labelled by non-Terminal symbols

Leaves: - Labelled by Terminal symbols or  $\epsilon$  (Epsilon)



## Context free Grammars

A grammar  $G = (V, T, P, S)$  is said to be a CFG if the productions of  $G$  are of the form:

$$A \rightarrow \alpha, \text{ where } \alpha \in (V \cup T)^*$$

The right hand side of a CFG is not restricted and it may be null or a

combination of variables and terminals. The possible length of right hand side can range from 0 to  $\infty$  i.e.,  $0 \leq | \alpha | < \infty$

As we know that a CFG has no context ~~dependency~~ neither left nor right. This is why, it is known as Context-free.



Ex:- Consider grammar  $G = (V, T, P, S)$  ②  
 having productions  $S \rightarrow asa \mid bsb \mid \epsilon$ .  
 check the productions and find the  
 language generated.

Solution:-  
 let  $P_1: S \rightarrow asa$  (RHS is terminal  
 variable terminal)

$P_2: S \rightarrow bsb$  (RHS is terminal  
 variable terminal)

$P_3: S \rightarrow \epsilon$  (RHS is null string)

since, all productions are of the form  
 $A \rightarrow \alpha$ , where  $\alpha \in (V \cup T)^*$ , hence

given grammar  $G$  is a CFG.

Language Generated:-  $S \rightarrow asa \text{ or } bsb$

(3)

$\Rightarrow a^1 a^1$  or  $b^1 b^1$

$\Rightarrow a^n a^n$  or  $b^n b^n$  (using n-step derivation)

$\Rightarrow a^n b^m b^m a^n$  or  $b^n a^m a^m b^n$  (using m-step derivation)

$\Rightarrow a^n b^m b^m a^n$  or  $b^n a^m a^m b^n$  (using  $S \rightarrow \epsilon$ )

$\therefore \text{So, } L(G) = \{ww^R : w \in (a+b)^*\}$

## ① Derivations from a Grammar

The set of all strings that can be derived from a Grammar is said to be the Language generated from that Grammar.

Ex! - ①

Consider the Grammar  $G_1 = (\{S, A\}, \{a, b\}, S,$

$$\{ \begin{array}{l} S \rightarrow aAb, \\ aA \rightarrow aaAb, \\ A \rightarrow \epsilon \end{array} \}$$

$$\begin{aligned} S &\rightarrow aAb \quad [\text{by } S \rightarrow aAb] \\ &\rightarrow aaAbb \quad [\text{by } aA \rightarrow aaAb] \\ &\rightarrow aaaaAbbb \quad [\text{by } aA \rightarrow aaAb] \\ &\rightarrow aaabbb \quad [\text{by } A \rightarrow \epsilon] \end{aligned}$$



Ex (2)

Grammar  $G_2 = \{ \{S, A, B\}, \{a, b\}, S, \{ S \rightarrow AB, A \rightarrow a, B \rightarrow b \} \}$

$S \rightarrow AB$   
 $\rightarrow ab$

Language generated by  $G_2 \Rightarrow L(G_2) = \{ab\}$

Ex (3)

$G_3 = \{ (S, A, B), \{a, b\}, S, \{ S \rightarrow AB, A \rightarrow aA, B \rightarrow bB \} \}$

(1)  $S \rightarrow AB$  [  $A \rightarrow a$   $B \rightarrow b$  ]  
 $\rightarrow ab$

(2)  $S \rightarrow AB$   
 $\rightarrow aA bB$   
 $\rightarrow aabb$

(3)  $S \rightarrow AB$   
 $\rightarrow aAb$   
 $\rightarrow aab$

(4)  $S \rightarrow AB$   
 $\rightarrow abB$   
 $\rightarrow abb$

depend on choice  
production we take  
we can generate  
string.

$$\downarrow L(G_3) = \{ ab, a^2b^2, a^2b, ab^2, \dots \} \text{ (3)}$$

$$L(G_3) = \{ a^m b^n \mid m \geq 0 \text{ and } n \geq 0 \}$$

This is the generalized way to generate the grammar  $G$ .

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# ① Language To Context free (Conversion)

Problem: - Show that the language  $L = \{a^m b^n \mid m \neq n\}$  is Context free

Solution: -

If it is possible to construct a CFG to generate this language then we say that the language is Context free.

Let us construct the CFG for the language defined.

Assume that  $m = n$

ie,  $n$  number of a's should be followed by  $n$  number of b's.

So, the CFG for this can be

$$S \rightarrow aSb / \epsilon \quad \text{--- ①}$$

But,  $L = \{a^m b^n \mid m \neq n\}$  means, a's should be followed by b's and number of a's should not be equal to number of b's.

② i.e,  $m \neq n$ .

Let us see the different cases when  $m > n$  and when  $m < n$ .

Case - 1 :-

$m > n$  :-

This case occurs if the number of a's more compared to number of b's. ~~and~~  
~~number of a's~~

The Extra a's can be generated using the production

$$A \rightarrow aA|a$$

and the extra a's generated from this production should be appended towards left of the string generated from the production shown in production 1.

This can be achieved by introducing one more production.

$$S_1 \rightarrow AS$$

③ So, even though from  $S$  we get  $n$  number of  $a$ 's followed by  $n$  number of  $b$ 's since it is preceded by a variable  $A$  from which we could generate extra  $a$ 's, number of  $a$ 's, ~~number of~~ followed by number of  $b$ 's are different.

Case 2<sup>o</sup>

$m < n$   $\rightarrow$

This case occurs if the number of  $b$ 's are more compared to number of  $a$ 's. The extra  $b$ 's can be generated using the production

$$B \rightarrow bB / b$$

and the extra  $b$ 's generated from this production should be appended towards right of the string generated from the production shown in production (1).



this can be achieved by introducing one more production. (4)

$$S_1 \rightarrow SB$$

The context free grammar  $G = (V, T, P, S)$  where

$$V = \{S_1, S, A, B\}, T = \{a, b\}$$

$$P = \{$$

$$S_1 \rightarrow AS \mid SB$$

$$S \rightarrow asb \mid \epsilon$$

$$A \rightarrow aA \mid a$$

$$B \rightarrow bB \mid b$$

}  $S_1$  is the start symbol.

generates the language  ~~$L = \{a^m b^n \mid m \neq n\}$~~

$$L = \{a^m b^n \mid m \neq n\}.$$

Since a CFG exists for the language.

∴ The language is context free.  
hence proved.

Problem:- Draw a CFG to generate a language consisting of equal number of a's and b's.

Solution:-

Note that initial production can be of the form

$$S \rightarrow aB/bA$$

If the first symbol is 'a', the second symbol should be a non-terminal from which we can obtain either 'b' or one more 'a' followed by two B's denoted by 'aBB' or a 'b' followed by S denoted by 'bS'.

Note that from all these symbols definitely we obtain equal number of a's and b's.

The productions corresponding to these can be of the form

$$B \rightarrow b/aBB/bS$$

on similar lines we can write  
A-productions as

(2)

$$A \rightarrow a | bAA | aS$$

from which we obtain a 'b' followed by

either

1. 'a' or

2. a 'b' followed by AA's denoted by

bAA or

3. Symbol 'a' followed by S denoted by aS

The context free grammar  $G = (V, T, P, S)$

where  $V = \{S, A, B\}$ ,  $T = \{a, b\}$

$$P = \{ S \rightarrow aB | bA$$

$$A \rightarrow aS | bAA | a$$

$$B \rightarrow bS | aBB | b$$

} S is the start symbol

generates the language consisting  
of equal number of a's and b's.

Hence proved



Problem: - obtain a CFG to generate integers.

Solution: -

The sign of a number can be '+' or '-' or  $\epsilon$ .

The production for this can be written as

$$S \rightarrow + | - | \epsilon$$

A number can be formed from any of the digits 0, 1, 2, ..., 9. The production to obtain these digits can be written as

$$D \rightarrow 0 | 1 | 2 | 3 | \dots | 9$$

A number  $N$  can be recursively defined as follows.

1. A number  $N$  is a digit  $D$  (i.e.,  $N \rightarrow D$ )
2. The number  $N$  followed by digit  $D$  is also a number (i.e.,  $N \rightarrow ND$ )

The production for this recursive definition can be written as

$$N \rightarrow D$$

$$N \rightarrow ND$$

An integer number  $I$  can be a number  $N$  or the sign  $S$  of a number followed by number  $N$ .

The production for this can be written as  $I \rightarrow N | SN$

So, the grammar  $G$  to obtain integer number can be written as  $G = \{V, T, P\}$  where  $V = \{D, S, N, I\}$

$$T = \{+, -, 0, 1, 2, 3, \dots, 9\}$$

$$P = \{I \rightarrow N | SN$$

$$N \rightarrow D | ND$$

$$S \rightarrow + | - | \epsilon$$

$$D \rightarrow 0 | 1 | 2 | 3 | \dots | 9$$

$\epsilon$   $S = \epsilon$  which is start symbol.

Hence Proved

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UNIT-3  
Continue

LEFTMOST AND RIGHTMOST DERIVATIONS

left most derivation:-

If  $G = (V, \Sigma, P, S)$  is a CFG and  $w \in L(G)$  then a derivation  $S \Rightarrow w$  is called leftmost derivation if and only if all steps involved in derivation have leftmost variable replacement only.

Right most derivation:-

If  $G = (V, \Sigma, P, S)$  is a CFG and  $w \in L(G)$  then a derivation  $S \Rightarrow w$  is called rightmost derivation if and only if all steps involved in derivation have rightmost variable replacement only.

Examples:-



Example 1 :- Consider the grammar

$S \rightarrow S + S / S * S | a | b$ . find leftmost

and rightmost derivations for string  $w$ .

ii,  $w = a * a + b$ .

Solution :-

leftmost derivation for  $w = a * a + b$

$S \Rightarrow S * S$  (using  $S \rightarrow S * S$ )

$\Rightarrow a * S$  (the first left hand symbol is  $a$   
so using  $S \rightarrow a$ )

$\Rightarrow a * S + S$  (using  $S \rightarrow S + S$ , in order to get  
 $a + b$ )

$\Rightarrow a * a + S$  (second symbol from the  
left is  $a$ , so using  $S \rightarrow a$ )

$\Rightarrow a * a + b$  (the last symbol from the  
left is  $b$ , using  $S \rightarrow b$ )

Right most derivation:-

for  $w = a * a + b$

$$S \xRightarrow[R]{} S * S \quad (\text{using } S \rightarrow S * S)$$

$$\xRightarrow[R]{} S * S + S \quad (\text{since, in the above sentential form second symbol is } * \text{ so we can not use } S \rightarrow a / b \text{ therefore, we use } S \rightarrow S + S)$$

$$\xRightarrow[R]{} S * S + b \quad (\text{using } S \rightarrow b)$$

$$\xRightarrow[R]{} S * a + b \quad (\text{using } S \rightarrow a)$$

$$\xRightarrow[R]{} a * a + b \quad (\text{using } S \rightarrow a)$$

Problem 2:-

Consider a CFG  $S \rightarrow bA | aB$ ,  $A \rightarrow as | aAA | a$ ,  
 $B \rightarrow bs | aBB | b$ . Find leftmost and rightmost  
derivations for  $w = a a a b b a b b b a$ .

Solution:-

leftmost derivation for  $w = a a a b b a b b b a$

$S \Rightarrow aB$  (using  $S \rightarrow aB$  to generate first symbol of  $w$ )

$\Rightarrow a a B B$  (since, second symbol is  $a$ , so we use  $B \rightarrow aBB$ )

$\Rightarrow a a a B B B$  (since, third symbol is  $a$ , so we use  $B \rightarrow aBB$ )

$\Rightarrow a a a b B B$  (since, fourth symbol is  $b$ , so we use  $B \rightarrow b$ )

$\Rightarrow a a a b b B$  (since, fifth symbol is  $b$ , so we use  $B \rightarrow b$ )



$\Rightarrow$  aaabbaBB (since first symbol is a, so we use  $B \rightarrow aBB$ )

$\Rightarrow$  aaabba**B** (since seventh symbol is b, so we use  $B \rightarrow b$ )

$\Rightarrow$  aaabba**bs** (since eighth symbol is b, so we use  $B \rightarrow bs$ )

$\Rightarrow$  aaabba**bs**A (since, ninth symbol is b, so we use  $S \rightarrow bA$ )

$\Rightarrow$  aaabba**bs**a (since, the tenth symbol is a, so using  $A \rightarrow a$ )

$\therefore S \Rightarrow$  aaabba**bs**a

Rightmost derivation  $\rightarrow$  for  $w =$  aaabba**bs**a

$S \Rightarrow$   aB  (using  $S \rightarrow aB$  to generate first symbol of  $w$ )

$\xrightarrow{R}$   aaBB  (we need a as the rightmost symbol and second symbol from the left side, so we use  $B \rightarrow aBB$ )

$\Rightarrow aABbs$  (we need ~~a~~ a as right most symbol and this is obtained from A only, we use  $B \rightarrow bs$ )

$\Rightarrow aaBbba$  (using  $S \rightarrow bA$ )

$\Rightarrow aaBbba$  (using  $A \rightarrow a$ )

$\Rightarrow aaaBBbba$  (we need b as the fourth symbol from the right)

$\Rightarrow aaaBbba$  (using  $B \rightarrow b$ )

$\Rightarrow aaa.bsbba$  (using  $B \rightarrow bs$ )

$\Rightarrow aaa.bbAbba$  (using  $S \rightarrow bA$ )

$\Rightarrow aaa.bba.bba$  (using  $A \rightarrow a$ )

R

Right most derivation  $S \xrightarrow[R]{} aaa.bba.bba$



problem 10-

(1)

Let  $G = (V, T, P, S)$  where  $V = \{s, c\}$ ,  $T = \{a, b\}$

$$P = \{ S \rightarrow aca$$

$$c \rightarrow aca \mid b$$

$S$  is the starting symbol.

What is the language generated by this grammar?

Solution:-

Consider the derivation

$$S \Rightarrow aca \Rightarrow aba \quad (\text{by applying 1}^{\text{st}} \& 3^{\text{rd}} \text{ production})$$

So, the string  $aba \in L(G)$

Consider the derivation

$$S \Rightarrow aca \quad \text{by applying } S \rightarrow aca$$

$$\Rightarrow aaca \quad \text{by applying } c \rightarrow aca$$

$$\Rightarrow aacaaa \quad \text{by applying } c \rightarrow aca$$



$\Rightarrow$  ---

(2)

$\Rightarrow$  ---

$\Rightarrow a^n c a^n$  by applying  $C \rightarrow aca$  (n times)

$\Rightarrow a^n b a^n$  by applying  $C \rightarrow b$

So, the Language  $L$  accepted by the

grammar  $G$  is  $L(G) = \{a^n b a^n \mid n \geq 1\}$

i.e, the Language  $L$  derived from the

grammar  $G$  is 'The string consisting of

$n$  number of  $a$ 's followed by a ' $b$ '

followed by  $n$  number of  $a$ 's.

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Problem 2:

what is the language generated by the grammar  $S \rightarrow OA \mid \epsilon$   
 $A \rightarrow IS$

Solution: The null string  $\epsilon$  can be obtained by applying the productions  $S \rightarrow \epsilon$  and the derivations is shown below

$S \Rightarrow \epsilon$  (By applying  $S \rightarrow \epsilon$ )

Consider the derivation

$S \Rightarrow OA$  (By applying  $S \rightarrow OA$ )

$\Rightarrow OIS$  (By applying  $A \rightarrow IS$ )

$\Rightarrow OIOA$  (By applying  $S \rightarrow OA$ )

$\Rightarrow OIOIS$  (By applying  $A \rightarrow IS$ )

$\Rightarrow OIOI$  (By applying  $S \rightarrow \epsilon$ )

② So, alternatively applying the productions  
 $S \rightarrow OA$  and  $A \rightarrow 1S$  and finally  
applying the production  $S \rightarrow \epsilon$ , we  
get string consisting of only 01's.

So, both null string i.e.  $\epsilon$   
and string consisting of 01's can be  
generated from this grammar.

So, The language generated by this  
grammar is

$$L = \{ w \mid w \in \{01\}^* \} \text{ or } L = \{ (01)^n \mid n \geq 0 \}$$



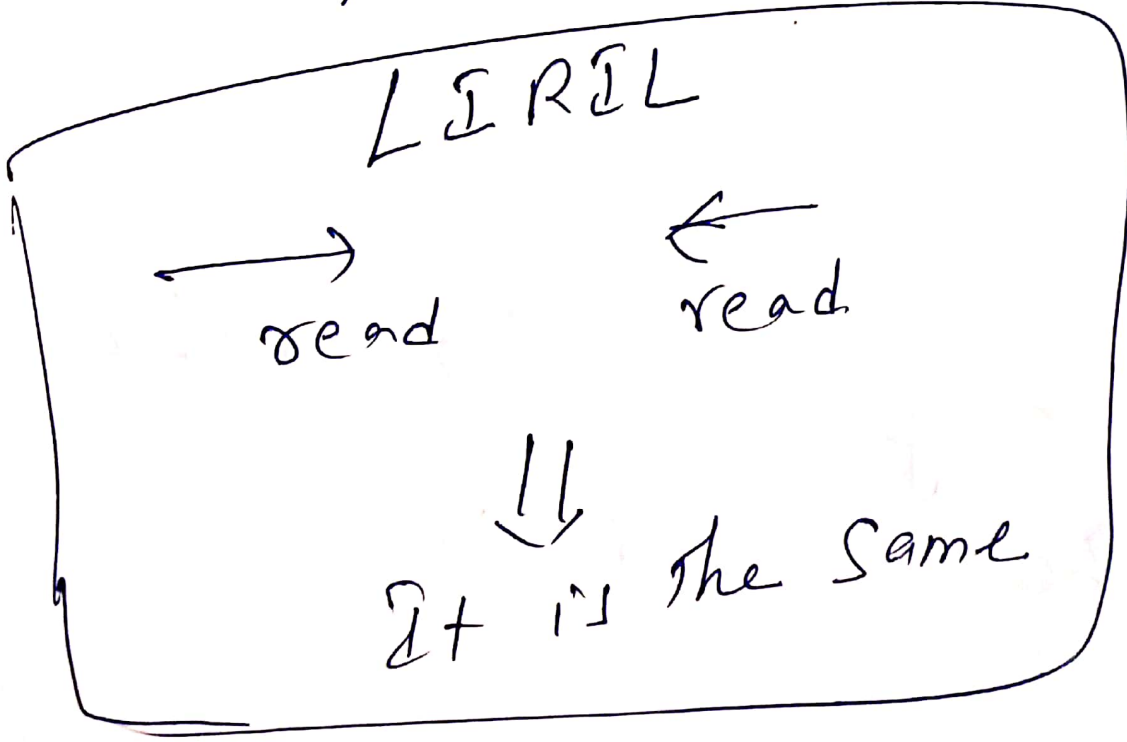
Problem 3:-

Construct CFG for the Language L which has all the strings which are all palindromes over  $T = \{a, b\}$

Solution:-

As we know the strings are palindromes if they possess same alphabets from forward as well as from backward

Ex:- Example string 'LIRIL' is Palindrome because



Since the language  $L$  is over  $T = \{a, b\}$  we want the production rules to be build a's and b's. As  $\epsilon$  can be the palindrome, a can be palindrome even b can be palindrome. So we can write the production rules as

$$G = (\{S\}, \{a, b\}, P, S)$$

$$P \text{ can be } S \rightarrow aSa$$

$$S \rightarrow bSb$$

$$S \rightarrow a$$

$$S \rightarrow b$$

$$S \rightarrow \epsilon$$

The string can be 'abaaba'  
will be derived as

3

$S \rightarrow aSa$

$\rightarrow abSba \quad (S \rightarrow \epsilon Sb)$

$\rightarrow abasa \quad (S \rightarrow aSa)$

$\rightarrow abaeaba \quad (S \rightarrow \epsilon)$

$\rightarrow abaaaba$

which is a palindrome

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## DERIVATION TREES

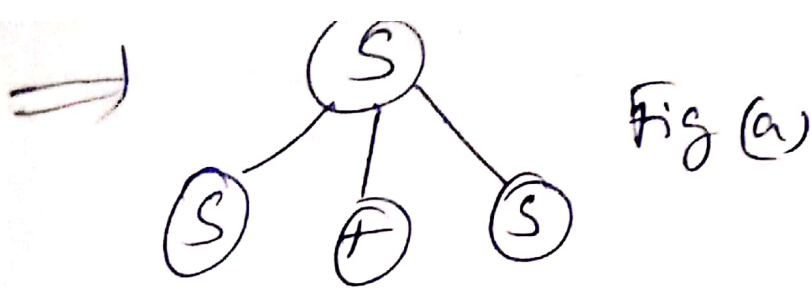
Let  $G = (V, T, P, S)$  is a CFG, Each production of  $G$  is represented with a tree satisfying the following conditions:

- ① If  $A \rightarrow \alpha_1 \alpha_2 \alpha_3 \dots \alpha_n$  is a production in  $G$ , then  $A$  becomes the parent of nodes labeled  $\alpha_1, \alpha_2, \alpha_3 \dots \alpha_n$  and
- ② The collection of children from left to right yields  $\alpha_1, \alpha_2, \alpha_3 \dots \alpha_n$

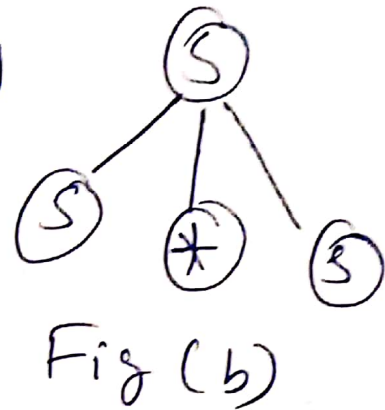
Example—

Consider a CFG  $S \rightarrow S+S | S * S | a | b$  and construct the derivation trees for all productions.

Solution— for production  $S \rightarrow S+S$ .



for production  $S \rightarrow S * S \Rightarrow$



For production  $S \rightarrow a \Rightarrow$

```

    graph TD
      S1((S)) --- a1((a))
  
```

fig (c)

for production  $S \rightarrow b \Rightarrow$

```

    graph TD
      S1((S)) --- b1((b))
  
```

fig (d)

If  $w \in L(a)$  then it is represented by a tree called derivation tree or parse tree satisfying the following conditions:

- ① The root has label  $S$  (the starting symbol)
- ② The all internal vertices (or nodes) are labeled with variables.
- ③ The leaves (or terminal) nodes are labeled with  $\in$  or terminal symbol.
- ④ If  $A \rightarrow \alpha_1 \alpha_2 \alpha_3 \dots \alpha_n$  is a production in  $G$  then  $A$  becomes the parent of nodes labeled  $\alpha_1 \alpha_2 \alpha_3 \dots \alpha_n$  and
- ⑤ ~~The~~ The collection of leaves from left to right yields the string  $w$ .

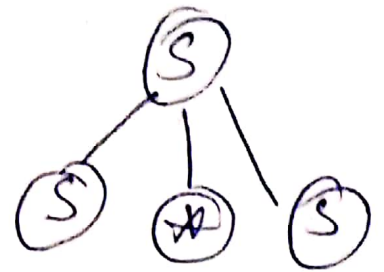
Example:- Consider the grammar  
 $S \rightarrow S + S \mid S * S \mid a \mid b$ . Construct  
 derivation tree for string  $w = a * b + a$ .

Solution:- The derivation tree or parse tree is shown as

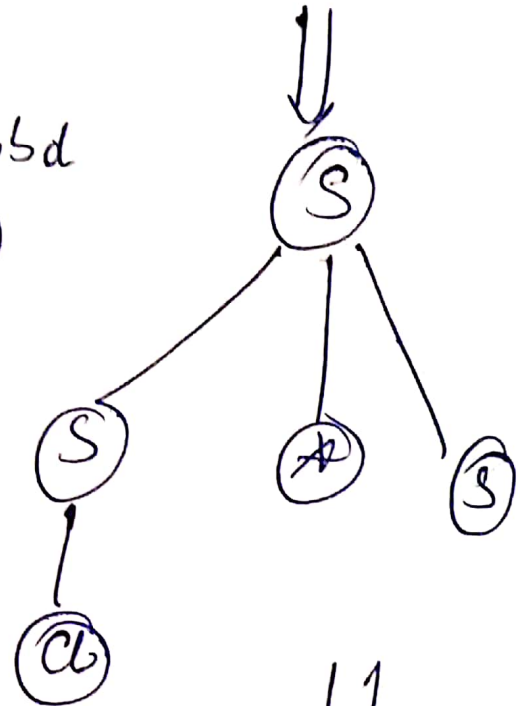


leftmost derivation: for  $w = a * b + a$ .

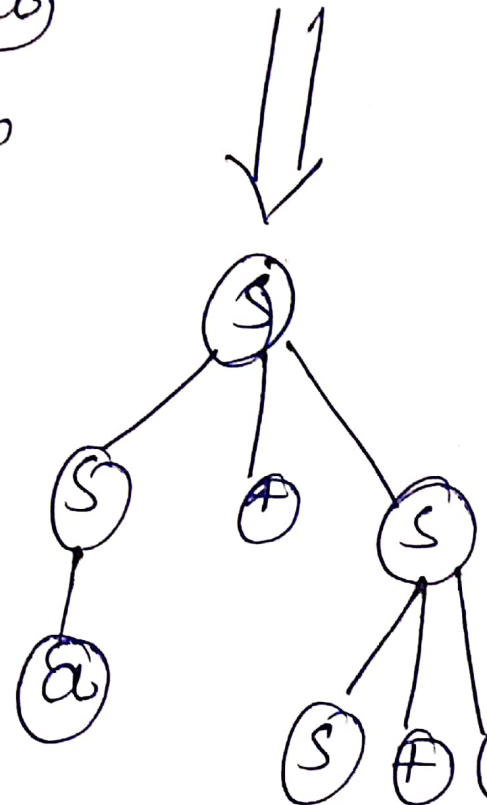
$S \Rightarrow S * S$  (using  $S \rightarrow S * S$ )  $\Rightarrow$



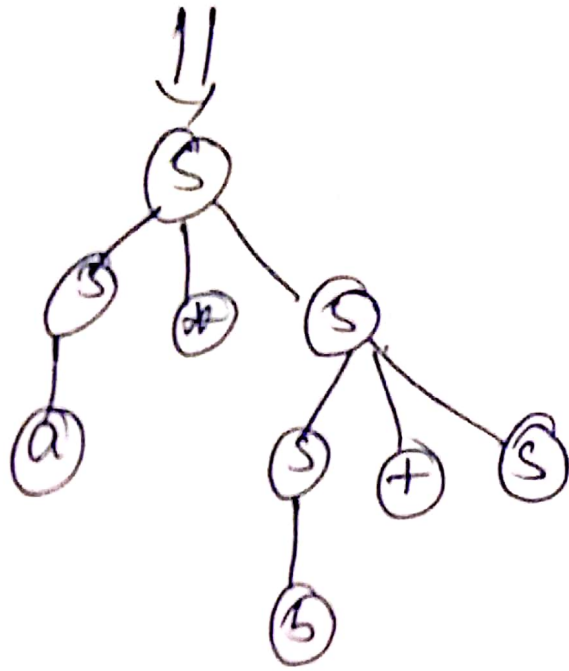
$\Rightarrow a * S$  (the first left hand symbol is  $a$ , so using  $S \rightarrow a$ )



$\Rightarrow a * S + S$  (using  $S \rightarrow S + S$ , in order to get  $b + a$ )



$\Rightarrow a * b + s$  (2<sup>nd</sup> symbol from left is  $b$ ,  
so using  $s \rightarrow b$ ).



$\Rightarrow a * b + a$  (The last symbol from the left  
is  $a$ , so using  $s \rightarrow a$ )

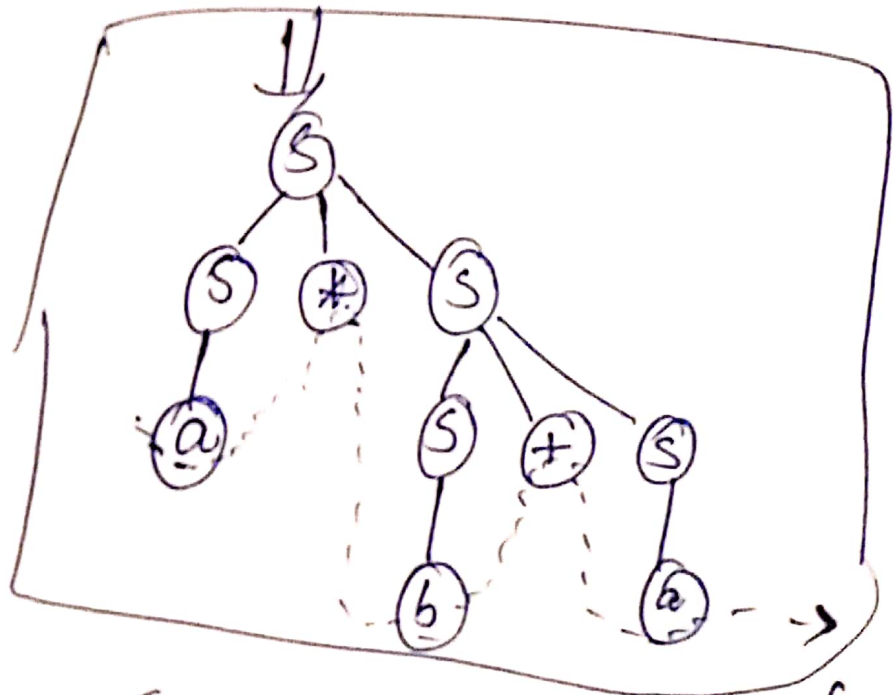


Figure of parse tree for  
 $a * b + a$ .

Example 2:-

Consider a grammar G having production  
 $S \rightarrow aAs/a$ ,  $A \rightarrow sBA/ss/ba$ .

Show that  $S \Rightarrow aabbba$  and construct  
a derivation tree whose yield is  $aabbba$ .

Solution:-

- $S \Rightarrow aAS$
- $\Rightarrow asbAS$
- $\Rightarrow aabAS$
- $\Rightarrow aabbas$
- $\Rightarrow aabbba$

Hence  $S \Rightarrow aabbba$   
Parse tree is shown  
in figure.

Parse Tree

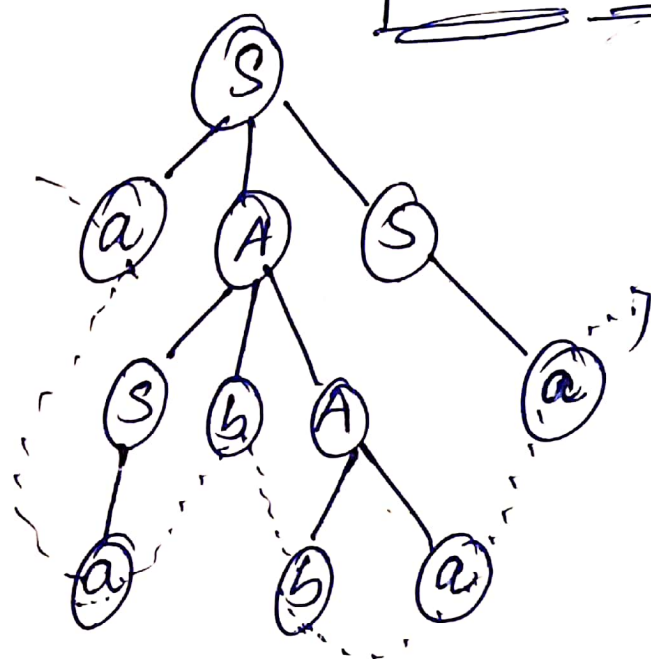


Fig: parse tree  
yielding  
 $aabbba$ .



Example 3 :- Consider the grammar  $G$   
whose productions are  
 $S \rightarrow 0B/1A$ ,  $A \rightarrow 0/0S/1AA$ ,  $B \rightarrow 1/1S/0BB$ .

Find (a) left most (2) Right most derivation  
for string 00110101 and Construct  
derivation tree also.

Solution :-

Left most derivation :-

$$S \Rightarrow 0B \Rightarrow 00BB$$

$$\Rightarrow 001B \Rightarrow 0011S$$

$$\Rightarrow 00110B \Rightarrow 001101S$$

$$\Rightarrow 0011010B \Rightarrow 00110101$$

Right most derivation

$$S \Rightarrow 0B \Rightarrow 00BB$$

$\Rightarrow 00131 \Rightarrow 00181$

$\Rightarrow 0011A1 \Rightarrow 0011051$

$\Rightarrow 001101A1 \Rightarrow 00110101$

Derivation tree.

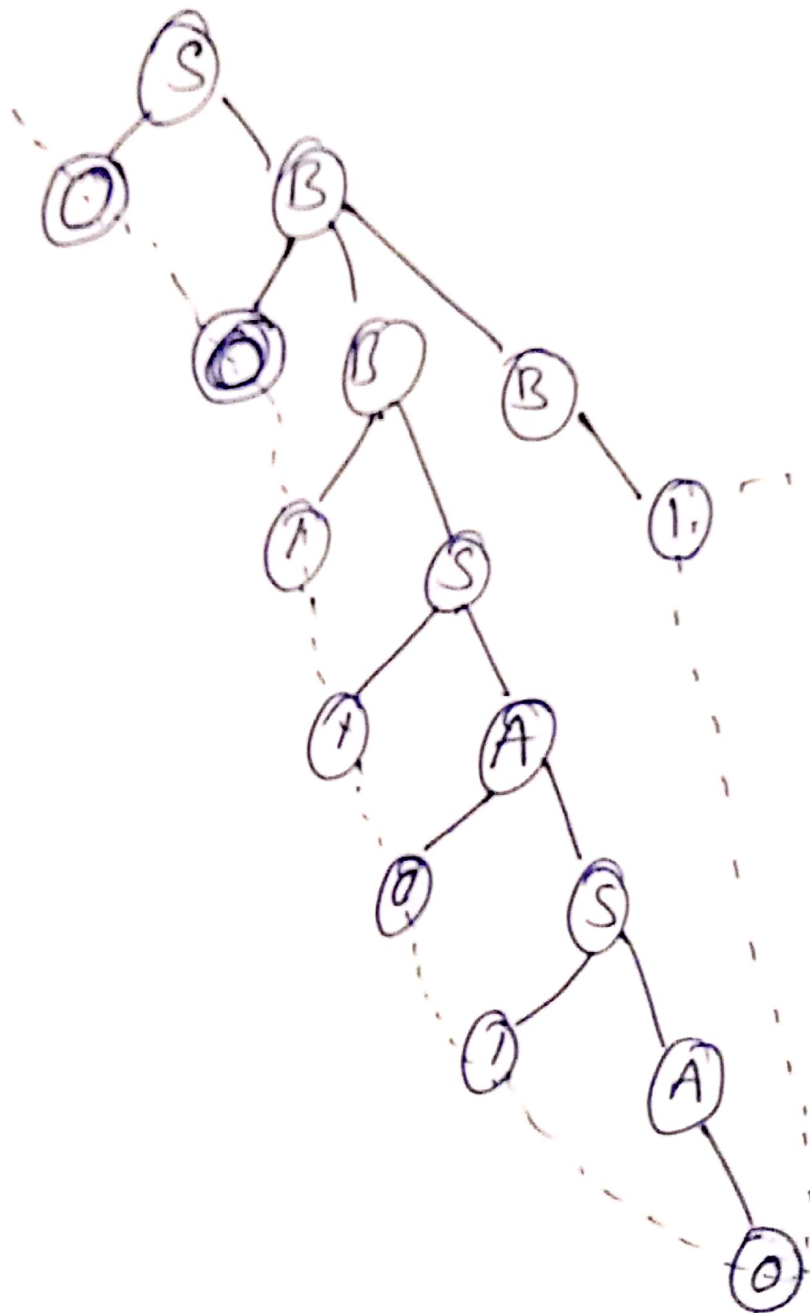


Fig.  $\Rightarrow 00110101$

is the derivation tree